Todd C. Atkins (SBN 208879)
tatkins@atkinsdavidson.com
ATKINS & DAVIDSON, APC
2261 Rutherford Road
Carlsbad, CA 92008
Tel: 619.665.3476

Matthew M. Wawrzyn (*pro hac vice* pending)
matt@wawrzynlaw.com
Stephen C. Jarvis (*pro hac vice* pending)
stephen@wawrzynlaw.com
WAWRZYN & JARVIS LLC
2700 Patriot Blvd, Suite 250
Glenview, IL 60026
Telephone: 847.656.5848

Attorneys for Nature Simulation Systems Inc.

UNITED STATES DISTRICT COURT

NORTHERN DISTRICT OF CALIFORNIA

NATURE SIMULATION SYSTEMS INC.,

Plaintiff,

v.

AUTODESK, INC.,

Defendant.

Case No.

COMPLAINT FOR PATENT INFRINGMENT

Parties

1. Plaintiff Nature Simulation Systems Inc. ("Nature Simulation") is a Quebec corporation with a place of business in Montreal, Quebec.

2. Autodesk, Inc. is a Delaware corporation with its principal place of business in San Rafael, California.

Jurisdiction and Venue

3. This action arises under the patent laws of the United States, 35 U.S.C. §§ 101 *et seq*.

4. This Court has subject matter jurisdiction over this action under 28 U.S.C. §§ 1331 and 1338(a).

5. This Court may exercise personal jurisdiction over Autodesk, which conducts continuous and systematic business and has headquarters located in California and this District. These patent infringement claims arise directly from Autodesk's continuous and systematic activity in this District. In short, this Court's exercise of jurisdiction over Autodesk would be consistent with the California long-arm statute and traditional notions of fair play and substantial justice.

6. Venue is proper in this District pursuant to 28 U.S.C. § 1400(b).

Claim of Patent Infringement

Count 1: U.S. Patent No. 10,120,961

7. Nature Simulation is the exclusive owner of United States Patent No. 10,120,961 (the "'961 patent"), which is attached hereto as "Exhibit A."

8. The '961 patent is valid and enforceable.

9. Autodesk has and is directly infringing claims of the '961 patent. Autodesk practices the methods embodied in the claims of the '961 patent. Without limiting the claims that

may be asserted or the services that may be accused of infringement in this action, Autodesk is infringing claim 1 of the '961 patent when Autodesk makes, uses, and sells its 3ds Max 2019 and 3ds Max 2020 software products.

a. Claim 1 is, "A method that performs immediate Boolean operations using geometric facets of geometric objects implemented in a computer system and operating with a computer, the method comprising . . . ." (Ex. A, col. 9:17-20.) 3ds Max is implemented in a computer system and operates with a computer. 3ds Max performs immediate Boolean operations using geometric facets of geometric objects. For example, 3ds Max renders a cylinder with different facets, where the Boolean operation yields variant results.

b. Claim 1 includes "mapping rendering facets to extended triangles that contain neighbors . . . ." (Ex. A, col. 9:21-22.) 3ds Max includes small planes formed from, for example, quads and triangles. When rendering the facets, neighboring information is *not* required; however, when conducting a Boolean operation, neighbors must be provided. 3ds Max uses B-Rep to indicate neighboring relationships.

c. Claim 1 continues, "building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection line by detecting whether two minimum bounding boxes overlap and performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection point

of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed . . . .” (Ex. A, col 9:23-33.) 3ds Max includes, for example, a Boolean intersection operation having two intersection lines, the top and bottom of a hexagon shape. In a second example, testing discloses that 3ds Max generates an intersection line that is a contour line, located in the middle section of a shaft shape. 3ds Max conducts edge-face calculations that equal edge-triangle intersection calculations. 3ds Max conducts real-time Boolean operations and surface trimming; each intersection line, tests disclose, is a closed curve when conducting a Boolean operation.

d. Claim 1 includes “splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end intersection points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion . . . .” (Ex. A, col. 9:34-40.) Testing of 3ds Max shows that all triangles and quads through which intersection lines pass are successively split. An example of this from testing of 3ds Max includes a triangle split into an upper, lower, and middle portion. Tests disclose that Delaunay mesh method has been implemented in 3ds Max systems. When performing a Boolean operation, the systems call the modified Watson method to ensure that each segment of an intersection line splits the facet. 3ds Max systems remove duplicate points through checking edge length when conducting a Boolean operation. A quad equals two triangles, and 3ds Max records a

quad with B-Rep data structure for Boolean operations. In other words, 3ds Max practices the splitting step, and a triangle or a quad gets a set of smaller triangles.

e. In claim 1, "checking each triangle whether it is obscure or visible for Boolean operations or for surface trimming . . . ." (Ex. A, 9:41-42.) 3ds Max successfully identifies each triangle as visible or obscure. Tests disclose, for example, a surface trimmed with a contour line, where each triangle is successfully classified.

f. Claim 1 continues, "regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets . . . ." (Ex. A, col. 9:43-47.) Tests disclose 3ds Max practicing the steps of copying and deleting triangles to form a new shape. For example, 3ds Max was found to delete a set of triangles inside the contour line when trimming the surface.

g. Claim 1 ends, "mapping extended triangles to rendering facets." (Ex. A, col. 9:48.) Tests disclose rendering facets that are quads, triangles, and polygons. 3ds Max conducts merging facets operations that lead to polygons. Tests disclose the Boolean operational result as quads, triangles, and polygons.

10. Autodesk is infringing claim 8 of the '961 patent when Autodesk makes, uses, and sells its 3ds Max software product.

a. Claim 8 is "The method of claim 1 wherein the checking each triangle whether it is obscure or visible when trimming a surface patch with a trimming contour

further composing . . . .” (Ex. A, col. 10:55-57.) 3ds Max includes surface trimming functions and conducts surface trimming with generic facets. Tests disclose a surface being trimmed with a contour, a circle, generating variant results; a surface is tessellated into facets rendered as quads and triangles. The facets through which a trimming contour passes are split into a set of triangles. These triangles and quads are classifies as obscure or visible while trimming.

b. Claim 8 continues, “setting m_ID of regular points of BIOpTriangleSet of the concerned patch to be 0 and m_ID of points of the intersection lines of the said patch in ascending or descending order, which is depending on whether the said line and the trimming contour are in the same direction . . . .” (Ex. A, col. 10:58-62.) Tests show that when trimming, each point of a geometric facets of a surface has an identifier to record its position. An intersection line is generated while trimming the surface patch with a trimming contour.

c. Claim 8 continues, “according to m_ID of the member m_Points of each triangle, deciding whether it is a boundary triangle . . . .” (Ex. A, col. 10:62-64.) In 3ds Max, triangles linked to an intersection line are boundary triangles.

d. Claim 8 concludes, “for each boundary triangle, determining it is to the left or right side of the trimming contour, and setting its neighbors that are not boundary ones to be left or right.” (Ex. A, col. 10:64-67.) 3ds Max systems have two modes for trimming a surface, which reserve the inside portion or the outside portion, corresponding to the left and the right side. 3ds Max systems link all triangles with edges and construct neighboring information.

Count 2: U.S. Patent No. 10,109,105

11. Nature Simulation is the exclusive owner of United States Patent No. 10,109,105 (the "'105 patent"), which is attached hereto as "Exhibit B."

12. The '105 patent is valid and enforceable.

13. Autodesk has and is directly infringing claims of the '105 patent. Autodesk practices the methods embodied in the claims of the '105 patent. Without limiting the claims that may be asserted or the services that may be accused of infringement in this action, Autodesk is infringing claim 1 of the '105 patent when Autodesk makes, uses, and sells its 3ds Max software product.

a. Claim 1 is, "A method that performs immediate Boolean operations using geometric facets of geometric objects implemented in a computer system and operating with a computer, the method comprising . . . ." (Ex. B, col. 8:47-50.) 3ds Max is implemented in a computer system and operates with a computer. 3ds Max performs immediate Boolean operations using geometric facets of geometric objects. For example, 3ds Max renders a cylinder with different facets, where the Boolean operation yields variant results.

b. Claim 1 includes "mapping rendering facets to extended triangles that contain neighbors . . . ." (Ex. B, col. 8:51-52.) 3ds Max includes small planes formed from, for example, quads and triangles. When rendering the facets, neighboring information is *not* required; however, when conducting a Boolean operation, neighbors must be provided. 3ds Max uses B-Rep to indicate neighboring relationships.

c. Claim 1 continues, "building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection

line by detecting whether two minimum bounding boxes overlap and performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection point of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed . . . .” (Ex. B, col 8:53-63.) 3ds Max includes, for example, a Boolean intersection operation having two intersection lines, the top and bottom of a hexagon shape. In a second example, testing discloses that 3ds Max generates an intersection line that is a contour line, located in the middle section of a shaft shape. 3ds Max conducts edge-face calculations that equal edge-triangle intersection calculations. 3ds Max conducts real-time Boolean operations; each intersection line, tests disclose, is a closed curve when conducting a Boolean operation.

d. Claim 1 includes “splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end intersection points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion . . . .” (Ex. B, col. 8:64-9:3.) Testing of 3ds Max shows that all triangles and quads through which intersection lines pass are successively split. An example of this from testing of 3ds Max includes a triangle split into an upper, lower, and middle portion. Tests disclose that Delaunay mesh method has been implemented in 3ds Max systems. When performing a Boolean operation, the systems call the modified

Watson method to ensure that each segment of an intersection line splits the facet. 3ds Max systems remove duplicate points through checking edge length when conducting a Boolean operation. A quad equals two triangles, and 3ds Max records a quad with B-Rep data structure for Boolean operations. In other words, 3ds Max practices the splitting step, and a triangle or a quad gets a set of smaller triangles.

e. In claim 1, "checking each triangle whether it is obscure or visible for Boolean operations . . . ." (Ex. B, 9:4-5.) 3ds Max successfully identifies each triangle as visible or obscure.

f. Claim 1 continues, "regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets . . . ." (Ex. B, col. 9:6-10.) Tests disclose 3ds Max practicing the steps of copying and deleting triangles to form a new shape. For example, 3ds Max was found to delete a set of triangles inside the contour line when generating the shaft shape.

g. Claim 1 ends, "mapping extended triangles to rendering facets." (Ex. B, col. 9:11.) Tests disclose rendering facets that are quads, triangles, and polygons. 3ds Max conducts merging facets operations that lead to polygons. Tests disclose the Boolean operational result as quads, triangles, and polygons.

Prayer for Relief

WHEREFORE, Nature Simulation prays for the following relief against Autodesk:

(a) Judgment that Autodesk has directly infringed the '961 patent and the '105 patent;

(b) For a fair and reasonable royalty;

(c) For pre-judgment interest and post-judgment interest at the maximum rate allowed by law;

(d) For such other and further relief as the Court may deem just and proper.

Demand for Jury Trial

Nature Simulation demands a trial by jury on all matters and issues triable by jury.

Date: June 7, 2019

/s/ Todd C. Atkins

Todd C. Atkins (SBN 208879)
tatkins@atkinsdavidson.com
ATKINS & DAVIDSON, APC
2261 Rutherford Road
Carlsbad, CA 92008
Tel: 619.665.3476

Matthew M. Wawrzyn (*pro hac vice* pending)
matt@wawrzynlaw.com
Stephen C. Jarvis (*pro hac vice* pending)
stephen@wawrzynlaw.com
WAWRZYN & JARVIS LLC
2700 Patriot Blvd, Suite 250
Glenview, IL 60026
Telephone: 847.656.5848

Attorneys for Nature Simulation Systems Inc.

EXHIBIT A

US010120961B2

(12) **United States Patent**
Cao

(10) **Patent No.: US 10,120,961 B2**
(45) **Date of Patent: *Nov. 6, 2018**

(54) **METHOD FOR IMMEDIATE BOOLEAN OPERATIONS USING GEOMETRIC FACETS**

(71) Applicant: **Shangwen Cao**, Montreal (CA)

(72) Inventor: **Shangwen Cao**, Montreal (CA)

(73) Assignee: **Nature Simulation Systems Inc.**, Montreal, Quebec (CA)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 0 days.

This patent is subject to a terminal disclaimer.

(21) Appl. No.: **15/840,052**

(22) Filed: **Dec. 13, 2017**

(65) **Prior Publication Data**

US 2018/0113958 A1 Apr. 26, 2018

Related U.S. Application Data

(63) Continuation-in-part of application No. 15/207,927, filed on Jul. 12, 2016.

(51) **Int. Cl.**

G06F 17/50	(2006.01)
G06T 17/10	(2006.01)
G06F 17/10	(2006.01)
G06T 17/20	(2006.01)
G06F 9/30	(2018.01)

(52) **U.S. Cl.**

CPC ***G06F 17/50*** (2013.01); ***G06F 9/30029*** (2013.01); ***G06F 17/10*** (2013.01); ***G06T 17/10*** (2013.01); ***G06T 17/20*** (2013.01); *G06T 2210/21* (2013.01)

(58) **Field of Classification Search**

CPC G06F 17/50

USPC 345/420

See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,825,369	A *	10/1998	Rossignac	G06T 9/40 345/440
5,886,702	A *	3/1999	Migdal	G06T 17/20 345/423
5,905,507	A *	5/1999	Rossignac	G06T 9/40 345/440
5,929,860	A *	7/1999	Hoppe	G06T 17/20 345/419

(Continued)

OTHER PUBLICATIONS

Landier, "Boolean operations on arbitrary polyhedral meshes", 24th International Meshing Roundtable (IMR24), 2015.

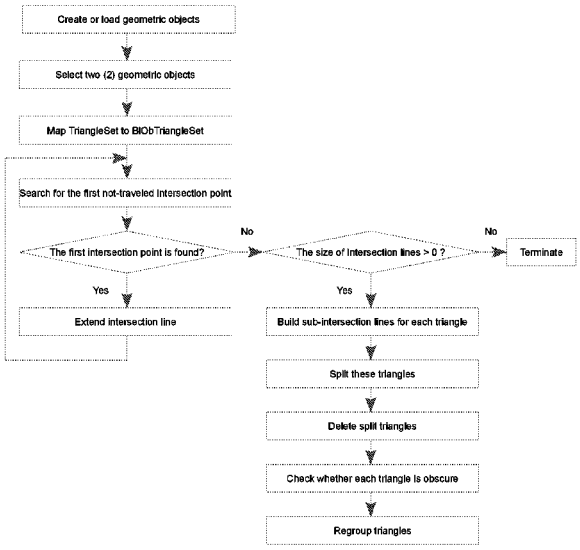
(Continued)

Primary Examiner — Jason C Olson

(57) **ABSTRACT**

A method for performing Boolean operations using a computer to create geometric models from primary geometric objects and their facets, comprises mapping rendering facets to extended triangles that contain neighbors; building intersection lines, splitting each triangle through which an intersection line passes, determining each facet is visible or obscure, and regrouping the facets to form one or more geometric objects. This method does not utilize the most popular data structures CSG and B-REP in CAD/CG/Solid Modeling systems, but has the advantages of both CSG and B-REP: easy to implement and flexible. Additionally it is a united method for solid modeling and surface modeling systems, and it is able to generate variant and editable models.

20 Claims, 6 Drawing Sheets



(56) **References Cited**

U.S. PATENT DOCUMENTS

5,945,996	A *	8/1999	Migdal	G06T 17/20 345/420
5,977,987	A *	11/1999	Duluk, Jr.	G06T 15/10 345/441
6,016,153	A *	1/2000	Gueziec	G06T 9/40 345/441
6,031,548	A *	2/2000	Gueziec	G06T 17/20 345/440
6,078,331	A *	6/2000	Pulli	G06T 17/20 345/423
6,172,679	B1 *	1/2001	Lim	G06T 15/40 345/421
6,191,787	B1 *	2/2001	Lu	G06T 17/05 345/418
6,307,555	B1 *	10/2001	Lee	G06T 17/20 345/421
6,573,895	B1 *	6/2003	Carter	G06T 15/40 345/423
6,587,105	B1 *	7/2003	Stam	G06T 17/20 345/423
6,850,638	B1 *	2/2005	Lounsbery	G06T 17/205 345/420
6,989,830	B2 *	1/2006	Stollnitz	G06T 17/20 345/420
7,127,380	B1 *	10/2006	Iverson	G06F 17/5018 703/2
7,133,043	B1 *	11/2006	Hollasch	G06T 15/06 345/421
7,209,137	B2 *	4/2007	Brokenshire	G06T 15/00 345/420
7,324,105	B1 *	1/2008	Moreton	G06T 17/20 345/420
7,366,581	B2 *	4/2008	Hill	G06T 17/10 700/118
7,408,553	B1 *	8/2008	Toksvig	G06T 11/40 345/423
7,589,720	B2 *	9/2009	Zhou	G06T 17/20 345/419
8,345,042	B2 *	1/2013	Kataoka	G06K 9/00214 345/420
8,547,395	B1 *	10/2013	Hutchins	G06T 11/40 345/611
9,401,046	B2 *	7/2016	Munkberg	G06T 17/205
9,721,363	B2 *	8/2017	Williams	G06T 11/203

OTHER PUBLICATIONS

Rappoport et al, "Interactive Boolean Operations for Conceptual Design of 3-D Solids", Institute of Computer Sciences, The Hebrew University, 1997.

E. Gursoz et al.,"Non-Regularized Boolean Set Operations on Non-Manifold B-Representation Objects", Computer-Aided Design 23 (1991) Jan./Feb. No. 1 London, GB.

D.F. Watson, "Computing the n-dimensional Delaunay tessellation with application to Voronoi polytopes", The Computer Journal 24 (2) 1981.

* cited by examiner

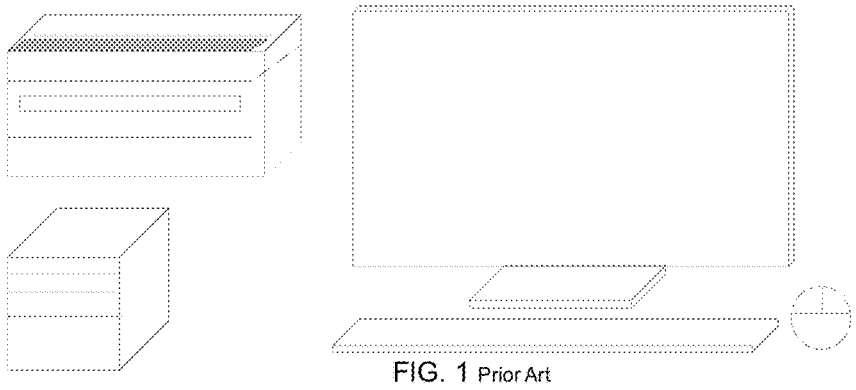


FIG. 1 Prior Art

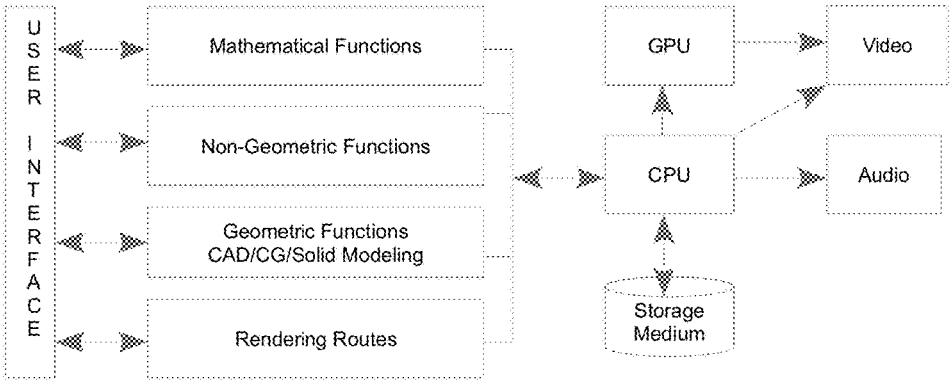


FIG. 2A Prior Art

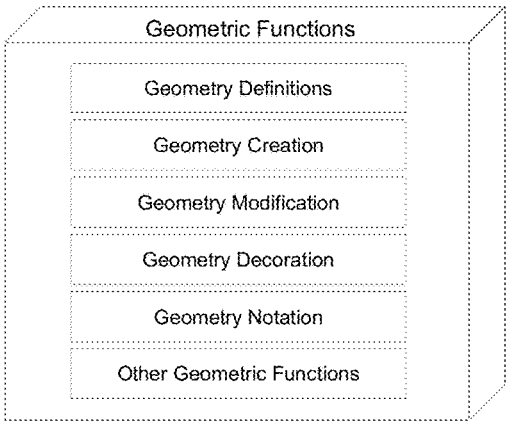


FIG. 2B Prior Art

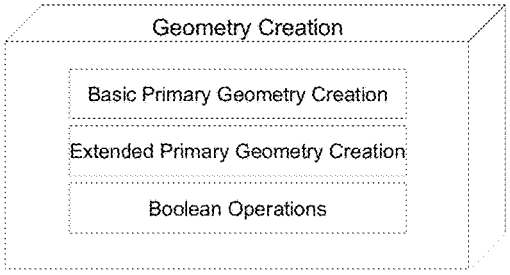


FIG. 2C Prior Art

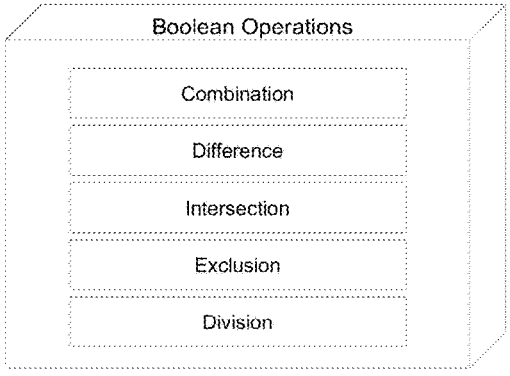


FIG. 2D Prior Art

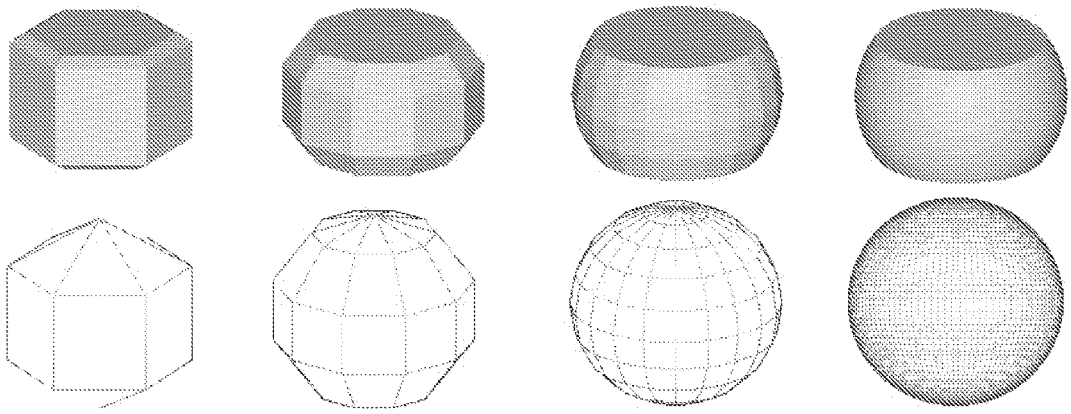


FIG. 3A Prior Art FIG. 3B Prior Art FIG. 3C Prior Art FIG. 3D Prior Art

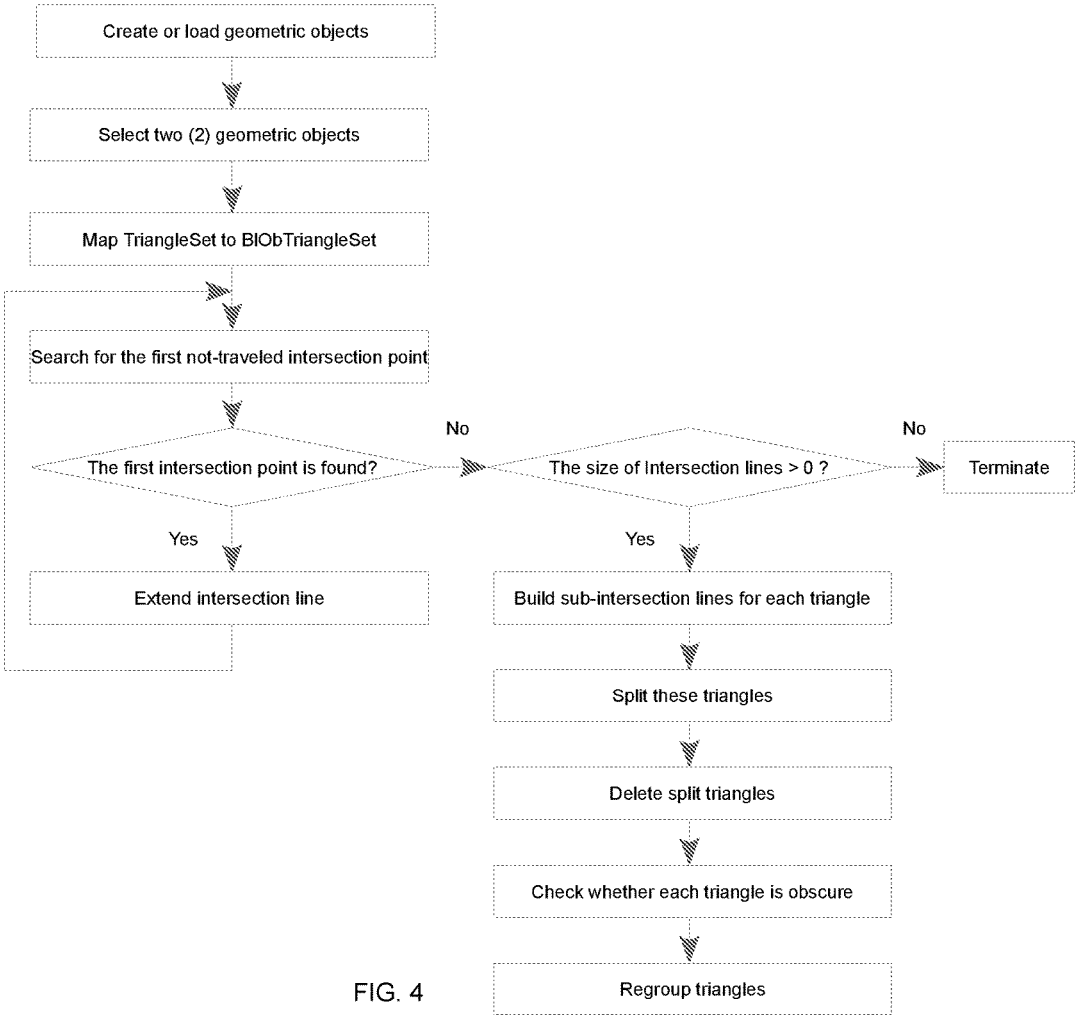


FIG. 4

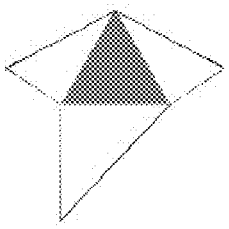


FIG. 5

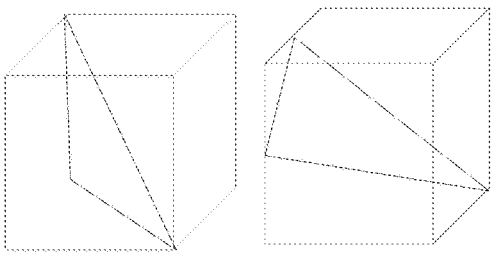


FIG. 6A

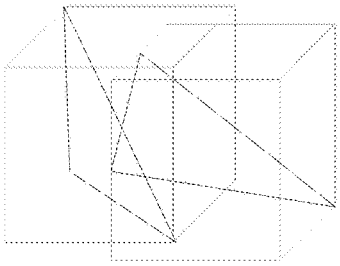


FIG. 6B

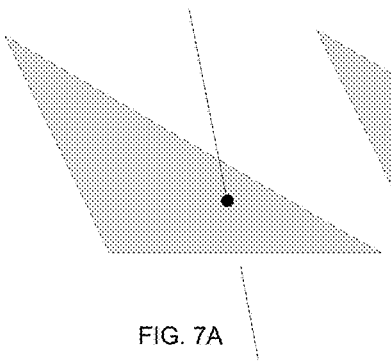


FIG. 7A

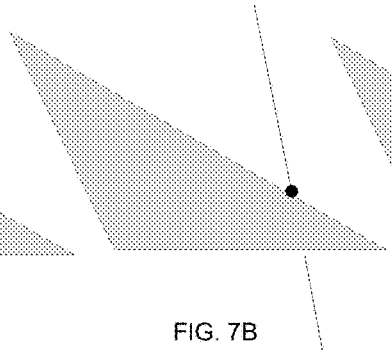


FIG. 7B

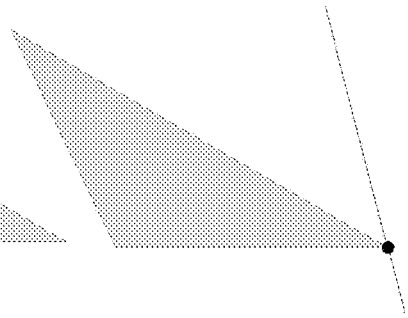


FIG. 7C

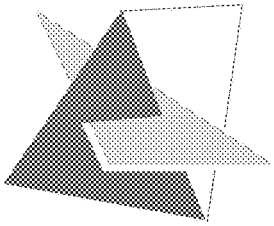


FIG. 8A

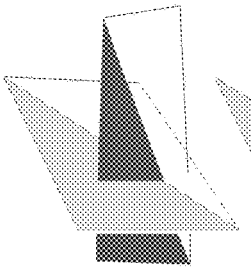


FIG. 8B

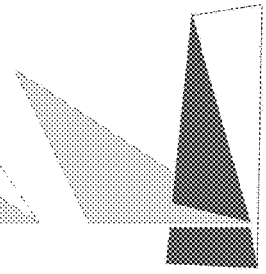


FIG. 8C

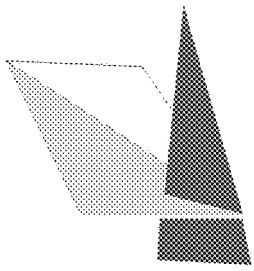


FIG. 8D

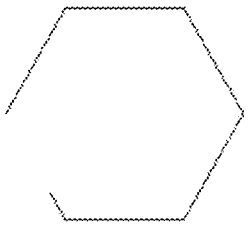


FIG. 9A

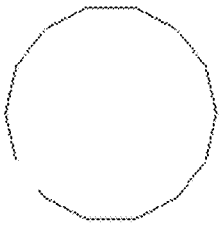


FIG. 9B

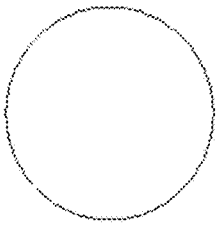


FIG. 9C

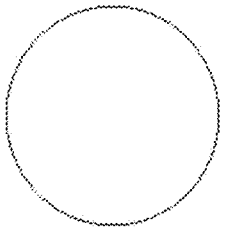


FIG. 9D

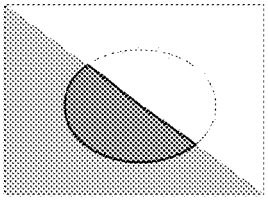


FIG. 10A

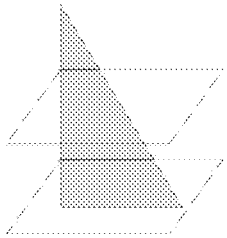


FIG. 10B

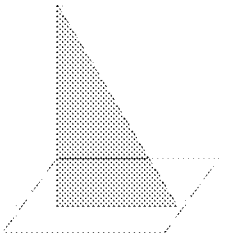


FIG. 10C

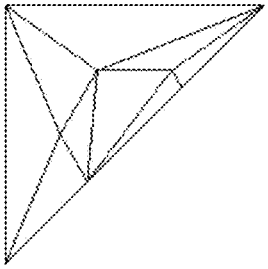


FIG. 11A

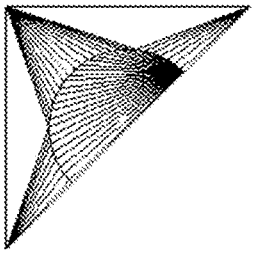


FIG. 11B

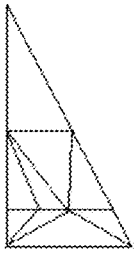


FIG. 11C

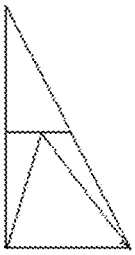


FIG. 11D

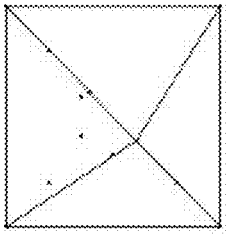


FIG. 12A Prior Art

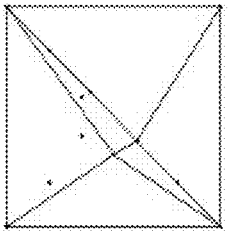


FIG. 12B Prior Art

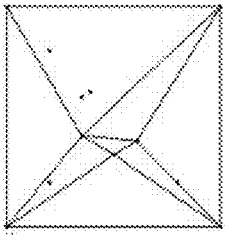


FIG. 12C Prior Art

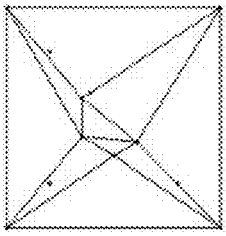


FIG. 12D Prior Art

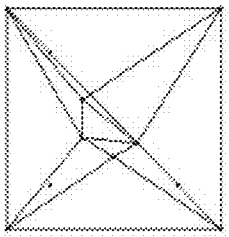


FIG. 12E Prior Art

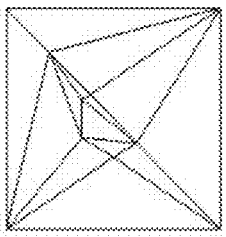


FIG. 12F Prior Art

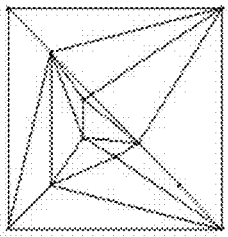


FIG. 12G

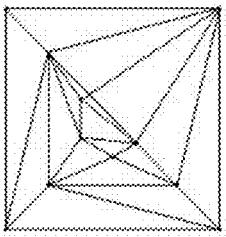


FIG. 12H

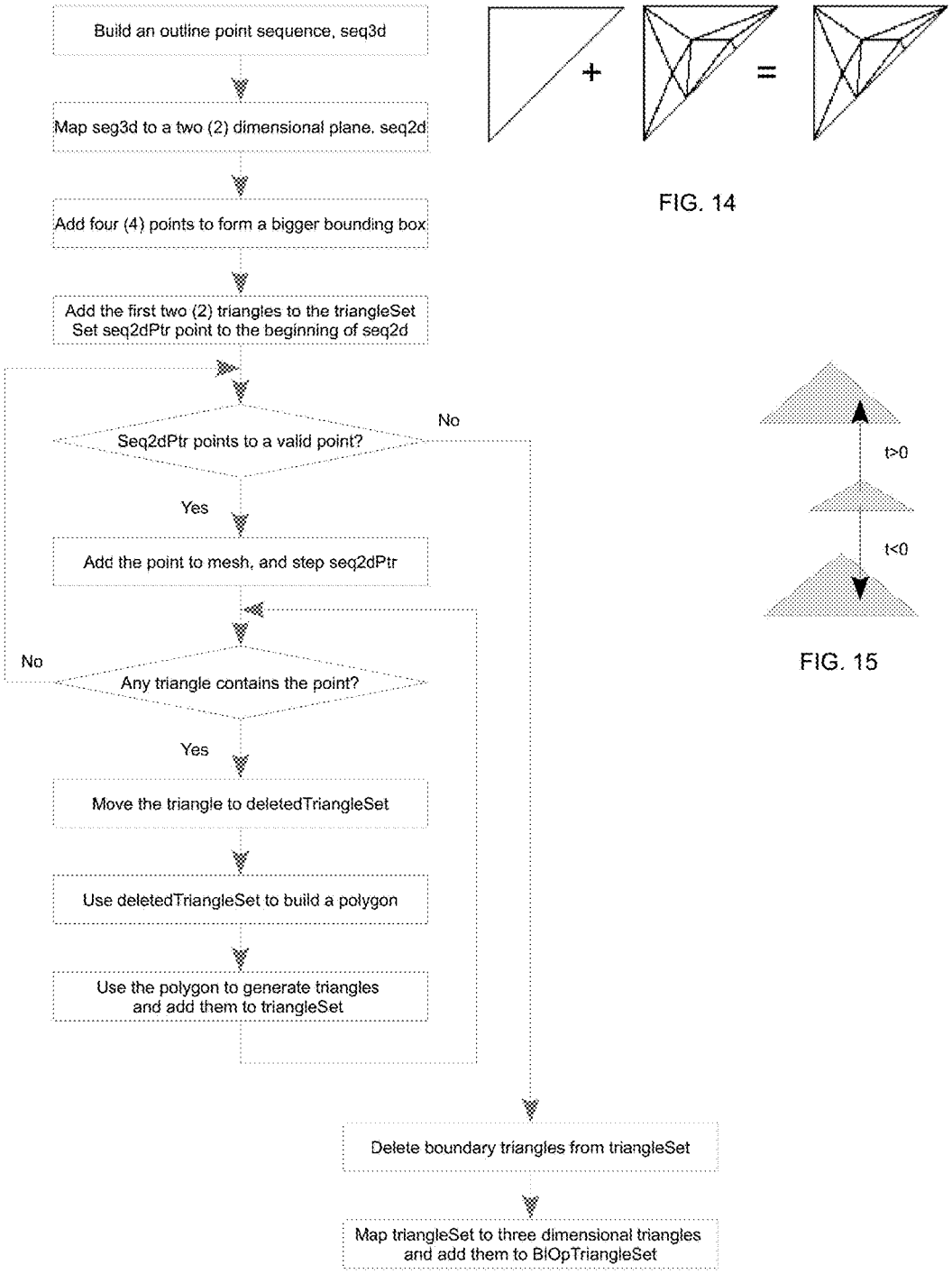


FIG. 13 Prior Art except the first two (2) steps , the last one, and the condition Any triangle contains the point.

FIG. 14

FIG. 15

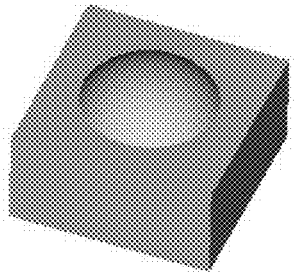


FIG. 16A

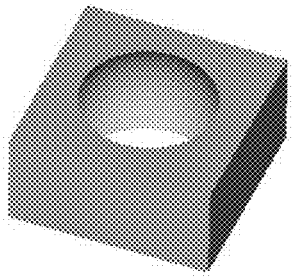


FIG. 16B

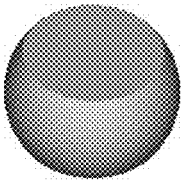


FIG. 16C

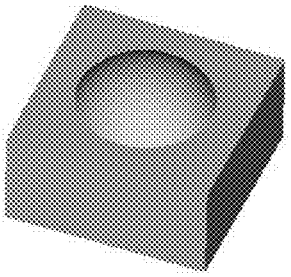


FIG. 16D

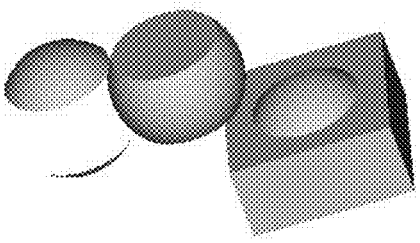


FIG. 16E

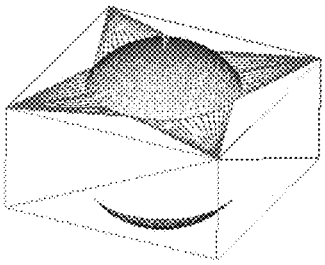


FIG. 16F

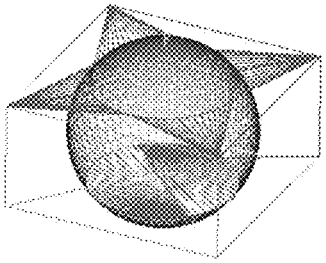


FIG. 16G

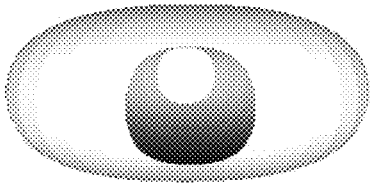


FIG. 17

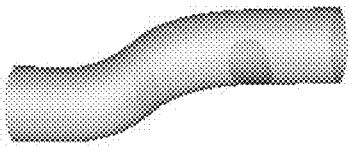


FIG. 18

METHOD FOR IMMEDIATE BOOLEAN OPERATIONS USING GEOMETRIC FACETS

BACKGROUND

Field of the Invention

This invention provides an immediate Boolean operation method for building three (3) dimensional geometric models from primary geometric objects to Computer Aided Design, Computer Graphics, Solid Modeling systems, and Surface Modeling systems, which are widely used in product design, manufacturing, and simulation. Mechanic industry, culture and sports, everywhere there are geometric shapes, may have CAD/CG applications.

Related Art

Computer hardware is so highly developed that even an ordinary Personal Computer may be used to install and run a commercial CAD/CG system, which normally has Boolean operation functions including AND, OR, and NOT. PC components comprise input devices, such as a mouse and a keyboard, a main machine, a screen, and a printer. The software system contains geometric and non geometric functions. FIG. **1** shows the main PC components and FIGS. **2**A through **2**D depict a typical CAD/CG software system architecture.

Boolean operations provide a general process of building complex solid geometric objects from different geometric shapes, which include primary geometric objects, swept or extruded objects, to CAD/CG/Solid Modeling systems. Lee applied Boolean operations to divide surface [Lee U.S. Pat. No. 6,307,555].

Boolean operations may rely on Constructive Solid Geometry, CSG, to record primary geometric objects and operation sequence in a hierarchic way, which technically is easy to implement, whereas Boundary Representation, B-REP, is regarded as a more flexible way that supports more geometric object types like extended geometries [Gursoz, 1990].

This invention presents five (5) Boolean operation commands: combination, intersection, exclusion, difference, and division, which directly work on triangles decomposed from geometric facets used for rendering functions and do not require the data structure Constructive Solid Geometry or Boundary Representation. The data structures defined in this invention are a few of simple classes, the algorithms incorporated in this invention are concise and easy to implement, and the five (5) commands allow the user to create geometric models not only by selecting the types of geometric objects but also by defining their facets. FIG. **3** presents that a box with 6 facets and a sphere with different facets make distinct results.

Although the five (5) commands are designed for solid modeling and surface modeling, surface trimming command is incorporated in this invention provides an alternative for surface modeling and it identifies whether a facet is obscure in a different way.

This invention presents data structures and algorithms differ from CSG and B-REP, and the algorithms incorporated in this invention include triangle-triangle intersections, splitting triangles with sub-intersection lines, identifying whether a facet is obscure, and regrouping triangles to form geometric models.

DISCLOSURE OF THE INVENTION

This invention provides a set of data structures and algorithms for performing Boolean operations, which are used to build complex geometric models and work directly on triangles decomposed from geometric facets used as rendering data by computer hardware and rendering functions like OpenGL libraries. A geometric shape, for example, a sphere, a cone, a cylinder, a box, triangular facets, an extruded or swept object, and a surface patch, is triangulated to build a set, noted as TriangleSet, for displaying. When two geometric shapes are selected for performing a Boolean operation, neighboring triangles will be added to each triangle in TriangleSet to form another set for each of the shapes, BIOpTriangleSet.

The second step of a Boolean operation this invention described is to search and build intersection lines between triangle sets. It starts with finding the first pair of intersecting triangles: this system builds an axis aligned minimum bounding box for each triangle and checks whether two bounding boxes overlap to decide if edge-triangle intersection needs to be calculated. Once the edge-triangle intersection point(s) falls inside a triangle, this system completes the searching task and stores the point data into an intersection line set.

To extend the current intersection line, this method traces neighboring triangles and calculates edge-triangle intersection points until the intersection line becomes closed.

The third step of a Boolean operation this invention described is to split triangles. Each segment of the intersection lines references two (2) triangles, each of the triangles has at least one sub-intersection line that contains one or more segments, which divide a triangle into three (3) or more smaller triangles. After splitting the triangles, the original triangles are removed, and those smaller triangles are added to the BIOpTriangleSet.

The fourth step of a Boolean operation this invention described is to decide each triangle is obscure or visible. If a triangle is enclosed by other triangles, it is obscure. A triangle is visible means it is outside another object.

The fifth step of a Boolean operation this invention described is to regroup the triangles: some of them have to be removed and some need to be put together, and there are five (5) cases for regrouping.

The final step of a Boolean operation this invention described is to map BIOpTriangleSet to TriangleSet.

The process of the said surface trimming command contains six (6) steps, too. Initially, this system maps a surface to a BIOpTriangleSet and one of its trimming contours to an extruded shape to form a BIOpTriangleSet. Step two (2), three (3), and six (6) are the same as that of the Boolean operations. Step four (4) checks a triangle is to the left or right side of the trimming contour to decide whether it is necessary to be reserved. The regrouping function, step five (5), deletes only left side or right side triangles when the system trims a surface.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** shows the main personal computer components, which generally contain a main machine, input devices including mouse and keyboard, a display, and a printer. A highly developed CAD/CG system can run on a PC machine.

FIGS. **2**A through **2**D describe a software architecture in which a CAD/CG/Geometric Modeling system uses Boolean operations and surface trimming to build geometric models.

FIG. **3** represents that distinct facets make various results even their original geometric object types and sizes are the same: the left side example has less facets and the right side

has more facets. In these examples, Boolean intersection operations work on a box and a sphere.

FIG. **4** is a flowchart for immediate Boolean operations using geometric facets.

FIG. **5** depicts that a triangle has three (3) neighbors. Given a triangle and its two vertices, there is one and only one neighboring triangle in solid models.

FIGS. **6**A and **6**B show two minimum bounding boxes do not overlap and two boxes overlap each other. Each triangle virtually has a minimum bounding box. If two boxes do not overlap, the triangles contained in the two boxes do not intersect. If the boxes overlap, edge-triangle intersection calculation is required.

FIGS. **7**A through **7**C depict three (3) edge-triangle intersection cases: an intersection point falls inside a triangle, an intersection point locates on an edge of a triangle, an intersection point is a vertex of a triangle.

FIGS. **8**A through **8**D show the searching candidate set, which allows the system to traverse next triangle for extending intersection lines by conducting edge-triangle calculation. Triangles filled with colors are the last pair of triangles that intersect each other, the triangles not filled are referenced by the member m_NeigTri of the data structure Triangle3dEx, which guides the system searching a minimum set of triangles when building intersection lines. The set contains one triangle, two triangles, or zero.

FIGS. **9**A through **9**D show four (4) examples of intersection lines. A box intersects a sphere, which has different facet numbers.

FIGS. **10**A through **10**C give three (3) examples of sub-intersection lines in darker color. FIG. **10**A has one (1) sub-intersection line, **10**B two (2), and **10**C one (1).

FIGS. **11**A through **11**D show four (4) examples that sub-intersection lines divided a triangle into a set of triangles.

FIGS. **12**A through **12**H show a Delaunay mesh sequence in which each intersection point is inserted into the mesh step by step.

FIG. **13** is the flowchart of Delaunay mesh modified Watson method that created the sequence of FIGS. **12**A through **12**H.

FIG. **14** shows a triangle and its Delaunay mesh. The original triangle is removed and only the Delaunay mesh is reserved for late computations.

FIG. **15** shows t-Buffer where t may be negative and positive. If the size of negative t and positive t is balanced in t-Buffer, the triangle concerned is enclosed by another object and is obscure.

FIGS. **16**A through **16**E show five (5) examples of Boolean operations conducted with a box and a sphere. FIGS. **16**F and **16**G depict the internal mesh of two Boolean operation resultants: combination and exclusion.

FIG. **17** shows a contour line trims a closed surface, a deformed sphere, and generates two (2) holes.

FIG. **18** gives an example in which an extruded surface, a tube, is trimmed by a contour line and creates a hole.

DETAILED DESCRIPTION

This invention defines these data structures: Point3dEx, Triangle3dEx, and BIOpTriangle3dSet that inherit Point3d, Triangle3d, Triangle3dSet storing facets for rendering geometric objects. When performing a Boolean operation, the system maps rendering facets to BIOpTriangle3dSet and all following processes focus on the members and attributes of BIOpTriangle3dSet. FIG. **4** is the flowchart describing the main procedure of Boolean operations conducted by the present invention. After a Boolean operation is completed, the system maps the resultant stored in BIOpTriangle3dSet to rendering facets.

Geometric Facets for Rendering

CAD systems render facets to represent a geometric object, such as a sphere, a cone, a box, a cylinder, an extruded or swept object. A facet may compose three (3) or more points, and facets are usually decomposed into triangles for easy calculations. A box has six (6) facets decomposed into twelve (12) triangles. A sphere may have eighteen (18) facets, composing twenty four (24) triangles. A sphere may also be rendered using more than one thousand (1,000) facets and triangles. FIG. **3** shows a sphere rendered with different facets. This method uses Triangle3dSet to note triangle set data structure for rendering a geometric object, it contains two (2) attributes: a three (3) dimensional point set and a triangle set, where Triangle3d references Point3d.

```
class Triangle3dSet
{
    DataSet<Point3d>    m_PointSet;
    DataSet<Triangle3d> m_TriangleSet;
};
class Triangle3d
{
    Point3d         *m_Points[3];
};
class Point3d
{
    DataTypeI m_X, m_Y, m_Z;
};
```

Triangles for Boolean Operations

The Boolean Operation method described in this invention defined three (3) key classes: BIOpTriangleSet, Triangle3dEx, and Point3dEx.

```
class BIOpTriangleSet
{
    DataSet<Point3dEx>          m_PointSet;
    DataSet<Triangle3dEx>       m_TriangleSet;
};
class Point3dEx : Point3d
{
        DataTypeII          m_ID; // position and sequence index
        DataTypeIII         m_X, m_Y, m_Z; // DataType III may
                            be different from DataTypeI
};
class Triangle3dEx : Triangle3d
{
        Point3dEx           *m_Points[3];
        DataTypeII           m_ID;
        Plane                m_Plane;
        DataTypeIV           m_Normal[3];
        Triangle3dEx        *m_NeigTri[3]; // neighboring triangles
};
```

DataTypeII may be int, long, unsigned long, or other integer types. DataTypeIII is a floating point data type, such as float, double, even long double.

The class Triangle3dEx specifies each triangle may have three (3) neighboring triangles, and every triangle is stored just one (1) copy in BIOpTriangleSet. Given the box example, the simplest way it has twelve (12) triangles, even each of them has three (3) neighbors, BIOpTriangleSet still stores a total of twelve (12) triangles.

Technically Triangle3d may have the attribute m_Normal. If DataTypeI and DataTypeIV are the same type, for example, double, the attribute m_Normal can be inherited.

Data Mapping

The process of mapping Triangle3dSet to BIOpTriangleSet copies point set and triangle set from rendering statue and fills default attributes. Data mapping contains the following procedure:

1) Copy points from Triangle3dSet to BIOpTriangleSet and ensure there are not identical points.
2) Copy triangles from Triangle3dSet to BIOpTriangleSet.
3) For each triangle in BIOpTriangleSet, set its neighboring triangles.
4) Calculate the normal and build the plane equation for each triangle in BIOpTriangleSet.

Remark 1: Given two (2) points a and b, if $|x_a-x_b|<\varepsilon$ and $|y_a-y_b|<\varepsilon$ and $z_a-z_b|<\varepsilon$, where ε is a positive floating point number, for example 5.0e-16, then b is identical to a.

Remark 2: When mapping points from rendering data to BIOpTriangleSet, the system checks if there is an identical point in BIOpTriangleSet.

Remark 3: A triangle, which has three (3) points, defines a plane whose mathematical formula is ax+by+cz+d=0 and the class Plane internally records it as an array of four (4) numbers, such as double m_ABCD[4].

Remark 4: A triangle, if its three (3) points are not identical, always has a valid normal. Even it is related to m_ABCD, a separate copy makes things more clear and easy to handle later.

Remark 5: Every triangle has three (3) edges, when there are no duplicated points, it has three (3) neighboring triangles in solid models. FIG. **5** shows an example: a triangle filled with dark color and its three (3) neighbors. When concerning a surface patch for surface trimming, one or two neighbors of a triangle may be null.

The First Intersection Point

Every triangle has three (3) vertices, which define a minimum bounding box. This method adopted the concept of axis aligned minimum bounding box.

Given a pair of triangles, if their bounding boxes do not overlap, the two triangles have no intersection point; otherwise, this method carries out edge-triangle intersection calculation.

If an edge of a triangle T_a intersects with a plane defined by a triangle T_b and the intersection point pet falls inside T_b, then pet is the first intersection point. If pet is outside of T_b, then switch the triangle position in the pair, (T_a, T_b) changed to (T_b, T_a), and conduct edge-triangle intersection calculation.

Given the i-th edge of a triangle T_a, i∈[0, 2], its formula is: $p=p_i+t*(p_{(i+1)\%3}-p_i)$, and the plane defined by the triangle T_b, its formula is: ax+by+cz+d=0. If the two formulas have a solution, the edge intersects with the plane. If the edge-plane intersection point falls inside the triangle T_b, then the point is the edge-triangle intersection point.

Extending an Intersection Line

This method defines a data structure for recording an intersection point as PntEgTri:

```
class PntEgTri
{
    Triangle3dEx      *m_Tri0, *m_Tri1;
    DataTypeII        m_EdgeIndex;
    DataTypeII        m_PointPosi;
    Point3dEx         m_Point;
    Point3dEx         *m_PntGlobalIndexA, *m_PntGlobalIndexB;
};
```

According to the location of an intersection point on a triangle, a PntEgTri, simply said pet, can be classified into three (3) categories shown in FIGS. **7**A through **7**C.

1) The most popular case is the edge-triangle intersection, pet locates on an edge of triangle T_a and inside triangle T_b.
2) Edge-edge intersection, pet locates on an edge of triangle T_a and on an edge of triangle T_b.
3) Edge-vertex intersection, pet locates on an edge of triangle T_a and on a vertex of triangle T_b.

To extend an intersection line, this system catches next neighboring triangle(s) and checks edge-triangle intersection until the intersection line gets closed or all triangles are traversed.

Sub-Intersection Line

An intersection line passes through a set of triangles and divides each triangle into multi partitions. The segments of an intersection line inside a triangle make up a sub-intersection line. FIGS. **10**A through **10**C show three (3) examples in which the dark lines are sub-intersection lines. In practice, a triangle may have zero (0), one (1), two (2) or three (3) sub-intersection lines.

The following algorithm shows how to get a valid reference to a triangle that has at least one sub-intersection line:

```
for each intersection line
  for each intersection point, get the triangle references: (m_Tri0, m_Tri1)
    for each triangle of the triangle pair, if it is not split
      for each intersection line
        search and build a sub-intersection line
```

Given a valid triangle and an intersection line, to decide if a pet belongs to the sub-intersection line of the triangle, this method checks whether

1) pet is on an edge of the triangle,
2) or pet is inside the triangle,
3) or pet equals a vertex of the triangle.

Splitting a Triangle

Given a set of sub-intersection lines, to split a triangle, this method

1) Removes duplicated pets. If neighboring pets are identical, this method reserves just one copy.
2) Identifies the position of end pets: checks each pet locates on which edge of the triangle.
3) Splits the upper partition, down partition, and middle partition of the triangle where applicable.

Given a set of points on a plane that represents a partition of a triangle, to decompose the plane into a group of triangles, this invention modified Delaunay 2D mesh Watson method, which is published in 1981 [Watson, 1981].

A Delaunay 2D mesh has three (3) data sets: triangle set that holds the generated triangles, deleted triangle set that

stores just deleted triangles, and polygon that records the outline of deleted triangle set.

The modified Delaunay 2D mesh method contains the following steps:

1) Build an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable.
2) Map the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane.
3) Add four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points.
4) Assume that one dialog line of the bounding box splits the box into two (2) triangles and add them into the triangle set.
5) Insert every point except bounding ones into the triangle set.
   a) For each point, check every triangle in the triangle set whether its circumcircle contains the point or the last segment passes through the triangle. If the condition is met, erase it from the triangle set and add it to the deleted triangle set.
   b) Use the deleted triangle set to extend the polygon and clear the deleted triangle set immediately.
   c) Use the polygon to generate triangles and add them to the triangle set.
6) Delete boundary triangles from the triangle set.
7) Map the triangle set to three dimensional triangles and add them to BIOpTriangleSet.

FIGS. **12**A through **12**H show a Delaunay 2D mesh sequence.

Deleting Split Triangles

In the above step, a split triangle got a mark. After all triangles have been traversed, this method deletes the marked triangles. FIG. **14** shows a deletion result.

Obscure Facets

Given two sets of triangles A and B, if A bounds a triangle of B, T_b, then T_b is obscure; if B bounds a triangle of A, T_a, then T_a is obscure.

To check whether a triangle T is bounded by an object O, this invention uses the following steps.

1) Calculate the centroid, c, of the triangle T
2) Build a line l: p=c+t*N, which starts from the centroid and passes along the normal N of the triangle T
3) For each triangle T_o of the object O, calculate line-plane intersection point. If there is a valid intersection point pet that falls inside the triangle T_o, then calculate t that is determined by centroid c and the pet, and add t to a depth buffer, butterT.
4) Check the size of negative t and positive t stored in butterT. If the two sizes are equal, then the triangle T is bounded and obscure.

When performing surface trimming, this system calls the followings procedure to determine whether a triangle is obscure.

1) Set the member m_ID of each Point3dEx of BIOpTriangleSet of the concerned surface patch to be 0.
2) Mark m_ID of Point3dEx of the intersection lines of the said patch in ascending or descending order, which is depending on whether the said line and trimming contour are in the same direction, for example, both of them are counterclockwise.
3) According to m_ID of the member m_Points of each triangle, decide whether it is a boundary triangle.
4) For each boundary triangle, decide it is to the left or right side of the trimming contour, and set its neighbors that are not boundary ones to be left or right.

Regrouping the Facets

This invention states five (5) kinds of Boolean operations, each of them has a different regrouping procedure.

The combination operation, logically it is OR, combines two solid geometric objects and generates a new object, which normally discards obscure partitions and reserves visible ones viewing from outside, has the following procedure.

1) Delete obscure triangles of object A.
2) Delete obscure triangles of object B.
3) Merge the triangles of object A and B.

The intersection operation, logically it is AND, which creates a solid geometric object using public sections of two geometric objects and discards any partitions of A and B outside the shared public sections, has the following procedure.

1) Delete NOT obscure triangles of object A.
2) Delete NOT obscure triangles of object B.
3) Merge the triangles of object A and B.

The exclusion operation, which builds a solid geometric object by removing public sections of two geometric objects and keeping not shared partitions, has the following procedure.

1) Copy object A's obscure triangles to a buffer, bufferA.
2) Delete obscure triangles from object A.
3) Copy object B's obscure triangles to object A.
4) Delete obscure triangles from object B.
5) Copy the triangles in bufferA to object B.
6) Reverse the normal of every obscure triangle of A and B.
7) Merge the triangles of the two objects.

The difference operation, which cuts geometric object A with another object B by removing any partitions of A inside B, has the following procedure.

1) Delete obscure triangles of object A.
2) Delete NOT obscure triangles of object B.
3) Reverse the normal of every triangle of object B.
4) Merge triangles of object A and B.

The division operation, which divides two solid geometric object A and B into three (3) objects, public sections of the two geometric objects, the NOT shared partitions of A and partitions of B, has the following procedure.

1) Copy object A's obscure triangles to a buffer, bufferA.
2) Copy object B's obscure triangles to bufferA.
3) Copy object A's obscure triangles to another buffer, bufferB.
4) Delete object A's obscure triangles.
5) Copy object B's obscure triangles to object A.
6) Delete object B's obscure triangles.
7) Copy object A's obscure triangles stored in bufferA to object B.
8) Reverse the normal of every obscure triangles of A and B.

Mapping to Rendering Facets

Once a Boolean operation is finished, this method maps BIOpTriangleSet to rendering triangles.

1) Each Point3dEx of BIOpTriangleSet is mapped to a Point3d of TriangleSet;

2) Each Triangle3dEx of BIOpTriangleSet is mapped to a Triangle3d of TriangleSet.

U.S. Patent Documents

U.S. Pat. No. 6,307,555 October 2001, Lee, 345/423.

Other Publications

"Non-Regularlized Boolean Set Operations on Non-Manifold B-Rep Objects", E. Gursoz et al., Carnegie Mellon University, Technical Report, 1990.

"Computing the n-dimensional Delaunay tessellation with application to Voronoi polytopes", D. F. Watson, The Computer Journal 24 (2) 1981.

What is claimed:

1. A method that performs immediate Boolean operations using geometric facets of geometric objects implemented in a computer system and operating with a computer, the method comprising:

mapping rendering facets to extended triangles that contain neighbors;

building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection line by detecting whether two minimum bounding boxes overlap and performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection point of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed;

splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end intersection points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion;

checking each triangle whether it is obscure or visible for Boolean operations or for surface trimming;

regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets; and

mapping extended triangles to rendering facets.

2. The method of claim **1** wherein any Boolean operations, that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets at the end of a Boolean operation or surface trimming without the data structure Constructive Solid Geometry and Boundary Representation.

3. The method of claim **1** wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets at the end of a Boolean operation or surface trimming with the data structure Constructive Solid Geometry or Boundary Representation.

4. The method of claim **1** wherein building intersection lines uses the minimum bounding boxes to detect whether two facets do not overlap and carries out edge-triangle intersection calculations comprising the steps ensuring that the intersection points are exact and the intersection lines are not approximate curves: building the formula $p=p_i+t*(p_{(i+1)\%3}-p_i)$ for expressing the i-th edge of the triangle T_a that is one triangle inside the triangle pair, building the formula $ax+by+cz+d=0$ for recording the plane defined by the triangle T_b that is another triangle inside the triangle pair, and getting the solution of the two linear formulas.

5. The method of claim **1** wherein searching for the first pair of triangles and searching neighboring triangles calculate edge-triangle intersection and employee neighboring triangles ensuring that direct calculation of edge-edge intersection is replaced by verifying whether a point is on an edge of a triangle.

6. The method of claim **1** wherein splitting each triangle and builds Delaunay 2D mesh with modified Watson method that defines a triangle set, a deleted triangle set, and a polygon, dividing the triangle into different partitions even when the sub-intersection lines are not convex, comprising of: building an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable; mapping the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane defined by the triangle; adding four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points; assuming that one dialog line of the bounding box splits the box into two (2) triangles and adding them into the triangle set; inserting every point except bounding ones into the triangle set with the steps: for each point, checking every triangle in the triangle set whether its circumcircle contains the point or the last segment passes through the triangle, and when the condition is met, erasing it from the triangle set and adding it to the deleted triangle set, using the deleted triangle set to extend the polygon and clearing the deleted triangle set immediately, and using the polygon to generate triangles and adding them to the triangle set; and deleting boundary triangles from the triangle set; mapping the triangle set to three dimensional triangles and adding them to BIOpTriangleSet.

7. The method of claim **1** wherein checking each triangle whether it is obscure or visible further composing: calculating the centroid c of triangle T_a that belongs to geometric object A; building a line l: $p=c+t*N$ passing through the centroid c and along the normal of T_a; for each triangle T_b of object B, checking whether l intersects with T_b at an interior point and adding t to a depth buffer, t-Buffer; and setting T_a to be obscure when the size of negative t equals to that of positive t in t-Buffer.

8. The method of claim **1** wherein the checking each triangle whether it is obscure or visible when trimming a surface patch with a trimming contour further composing: setting m_ID of regular points of BIOpTriangleSet of the concerned patch to be 0 and m_ID of points of the intersection lines of the said patch in ascending or descending order, which is depending on whether the said line and the trimming contour are in the same direction; according to m_ID of the member m_Points of each triangle, deciding whether it is a boundary triangle; for each boundary triangle, determining it is to the left or right side of the trimming contour, and setting its neighbors that are not boundary ones to be left or right.

9. The method of claim 1 wherein a Boolean operation that is a combination, an intersection, an exclusion, a difference, or a division, regroups facets for constructing its operational result using one or more steps of: deleting obscure or visible triangles of an object, copying obvious triangles of an object to a buffer or copying triangles from a buffer to an object, reversing the normal of each triangle of an object, and merging the triangles of the objects to form new extended triangle sets.

10. The method of claim 1 wherein the extended triangles are directly mapped to rendering facets for being displayed and providing data to next Boolean operations.

11. A computer system consisting of hardware and software that performs immediate Boolean operations using rendering facets of geometric objects, the system comprising:

a computer with input devices for entering data and commands, and a display device showing user interface, geometric objects, and additional data, having a medium storing geometric data and instructions that make up of a software system, or having a microchip or integrated circuit embedding partially or totally the instructions, and a processor that executes the steps of:

creating, modifying or loading primary geometric objects including swept and extruded ones and relocating them at different positions or orientations with input devices of the computer;

selecting two of the geometric objects;

mapping rendering facets to extended triangles that contain neighbors;

building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection line by detecting whether two minimum bounding boxes overlap and by performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection point of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed;

splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end intersection points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion;

checking each triangle whether it is obscure or visible for Boolean operations or for surface trimming;

regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets; and

mapping extended triangles to rendering facets.

12. The system of claim 11 wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets at the end of a Boolean operation or surface trimming with the data structure Constructive Solid Geometry and Boundary Representation.

13. The system of claim 11 wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets at the end of a Boolean operation or surface trimming with the data structure Constructive Solid Geometry or Boundary Representation.

14. The system of claim 11 wherein searching for the first pair of triangles that hold a start point of an intersection line and searching neighboring triangles of the last triangle pair that hold the last intersection point composed the procedure for building an intersection line that usually repeats more than one time when building intersection lines use the minimum bounding boxes to detect whether two facets do not overlap and carries out edge-triangle intersection calculations comprising the steps ensuring that the intersection points are exact and the intersection lines are not approximate curves: building the formulae $p=p_i+t*(p_{(i+1)\%3}-p_i)$ for expressing the i-th edge of the triangle T_a that is one triangle inside the triangle pair, building the formula $ax+by+cz+d=0$ for recording the plane defined by the triangle T_b that is another triangle inside the triangle pair, and getting the solution of the two linear formulas.

15. The system of claim 11 wherein searching for the first pair of triangles and searching neighboring triangles calculate edge-triangle intersection and employee neighboring triangles ensuring that direct calculation of edge-edge intersection is replaced by verifying whether a point is on an edge of a triangle.

16. The system of claim 11 wherein splitting each triangle builds Delaunay 2D mesh with modified Watson method that defines a triangle set, a deleted triangle set, and a polygon, dividing the triangle into different partitions even when the sub-intersection lines are not convex, comprising of: building an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable; mapping the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane defined by the triangle; adding four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points; assuming that one dialog line of the bounding box splits the box into two (2) triangles and adding them into the triangle set; inserting every point except bounding ones into the triangle set with the steps: for each point, checking every triangle in the triangle set whether its circumcircle contains the point or the last segment passes through the triangle, and when the condition is met, erasing it from the triangle set and adding it to the deleted triangle set, using the deleted triangle set to extend the polygon and clearing the deleted triangle set immediately, and using the polygon to generate triangles and add them to the triangle set; and deleting boundary triangles from the triangle set; mapping the triangle set to three dimensional triangles and adding them to BIOpTriangleSet.

17. The system of claim 11 wherein checking each triangle whether it is obscure or visible further composing: calculating the centroid c of triangle T_a that belongs to geometric object A; building a line l: $p=c+t*N$ passing through the centroid c and along the normal of T_a; for each triangle T_b of object B, checking whether l intersects with T_b at an interior point and adding t to a depth buffer, t-Buffer; and setting T_a to be obscure when the size of negative t equals to that of positive t in t-Buffer.

18. The system of claim **11** wherein checking each triangle whether it is obscure or visible when trimming a surface patch with a trimming contour further composing: setting m_ID of regular points of BIOpTriangleSet of the concerned patch to be 0 and m_ID of points of the intersection lines of the said patch in ascending or descending order, which is depending on whether the said line and the trimming contour are in the same direction; according to m_ID of the member m_Points of each triangle, deciding whether it is a boundary triangle; for each boundary triangle, determining it is to the left or right side of the trimming contour, and setting its neighbors that are not boundary ones to be left or right.

19. The system of claim **11** wherein a Boolean operation that is a combination, an intersection, an exclusion, a difference, or a division, regroups facets for constructing its operational result using one or more steps of: deleting obscure or visible triangles of an object, copying obscure triangles of an object to a buffer or copying triangles from a buffer to an object, reversing the normal of each triangle of an object, and merging the triangles of the objects to form new extended triangle sets.

20. The system of claim **11** wherein the extended triangles are directly mapped to rendering facets for being displayed and providing data to next Boolean operations.

* * * * *

EXHIBIT B

US010109105B2

(12) **United States Patent**
Cao

(10) **Patent No.: US 10,109,105 B2**
(45) **Date of Patent: Oct. 23, 2018**

(54) **METHOD FOR IMMEDIATE BOOLEAN OPERATIONS USING GEOMETRIC FACETS**

(71) Applicant: **Shangwen Cao**, Montreal (CA)

(72) Inventor: **Shangwen Cao**, Montreal (CA)

(73) Assignee: **Nature Simulation Systems Inc.**, Montreal, Quebec (CA)

(*) Notice: Subject to any disclaimer, the term of this patent is extended or adjusted under 35 U.S.C. 154(b) by 50 days.

(21) Appl. No.: **15/207,927**

(22) Filed: **Jul. 12, 2016**

(65) **Prior Publication Data**

US 2018/0018818 A1 Jan. 18, 2018

(51) **Int. Cl.**
G06T 17/10 (2006.01)
G06T 15/04 (2011.01)

(52) **U.S. Cl.**
CPC ***G06T 17/10*** (2013.01); ***G06T 15/04*** (2013.01); *G06T 2210/12* (2013.01)

(58) **Field of Classification Search**
CPC G06F 17/50
USPC 345/420
See application file for complete search history.

(56) **References Cited**

U.S. PATENT DOCUMENTS

5,825,369 A * 10/1998 Rossignac G06T 9/40 345/440
5,886,702 A * 3/1999 Migdal G06T 17/20 345/423
5,905,507 A * 5/1999 Rossignac G06T 9/40 345/440
5,929,860 A * 7/1999 Hoppe G06T 17/20 345/419
5,945,996 A * 8/1999 Migdal G06T 17/20 345/420
5,977,987 A * 11/1999 Duluk, Jr. G06T 15/10 345/441
6,016,153 A * 1/2000 Gueziec G06T 9/40 345/441
6,031,548 A * 2/2000 Gueziec G06T 17/20 345/440
6,078,331 A * 6/2000 Pulli G06T 17/20 345/423
6,172,679 B1 * 1/2001 Lim G06T 15/40 345/421
6,191,787 B1 * 2/2001 Lu G06T 17/05 345/418
6,307,555 B1 * 10/2001 Lee G06T 17/20 345/421
6,573,895 B1 * 6/2003 Carter G06T 15/40 345/423

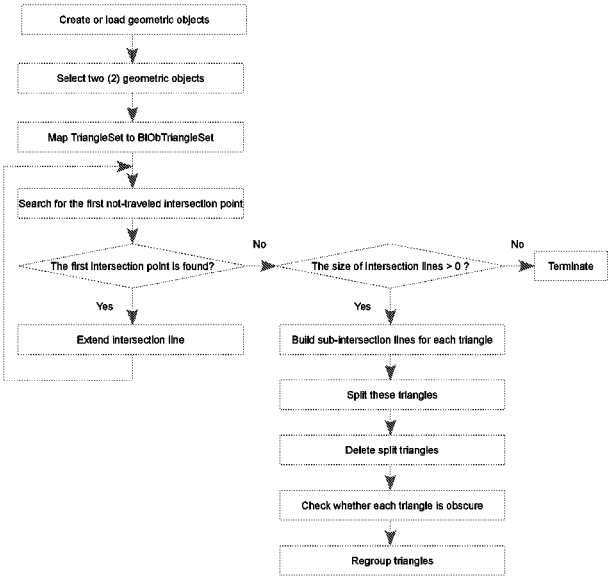
(Continued)

Primary Examiner — Jason C Olson

(57) **ABSTRACT**

A method for performing Boolean operations using a computer to create geometric models from primary geometric objects and their facets, comprises mapping rendering facets to extended triangles that contain neighbors, building intersection lines, splitting each triangle through which an intersection line passes, determining each triangle is obscure or visible, and regrouping the triangles to form one or more geometric objects. This method does not utilize the most popular data structures CSG and B-REP in CAD/CG/Solid Modeling systems, but has the advantages of both CSG and B-REP: easy to implement and so flexible that it can handle concave and convex geometric shapes, swept and extruded geometric objects, and it is able to generate variant and editable models.

20 Claims, 6 Drawing Sheets



(56) **References Cited**

U.S. PATENT DOCUMENTS

6,587,105	B1 *	7/2003	Stam	G06T 17/20 345/423
6,850,638	B1 *	2/2005	Lounsbery	G06T 17/205 345/420
6,989,830	B2 *	1/2006	Stollnitz	G06T 17/20 345/420
7,127,380	B1 *	10/2006	Iverson	G06F 17/5018 703/2
7,133,043	B1 *	11/2006	Hollasch	G06T 15/06 345/421
7,209,137	B2 *	4/2007	Brokenshire	G06T 15/00 345/420
7,324,105	B1 *	1/2008	Moreton	G06T 17/20 345/420
7,366,581	B2 *	4/2008	Hill	G06T 17/10 700/118
7,408,553	B1 *	8/2008	Toksvig	G06T 11/40 345/423
7,589,720	B2 *	9/2009	Zhou	G06T 17/20 345/419
8,345,042	B2 *	1/2013	Kataoka	G06K 9/00214 345/420
8,547,395	B1 *	10/2013	Hutchins	G06T 11/40 345/611
9,401,046	B2 *	7/2016	Munkberg	G06T 17/205
9,721,363	B2 *	8/2017	Williams	G06T 11/203

* cited by examiner

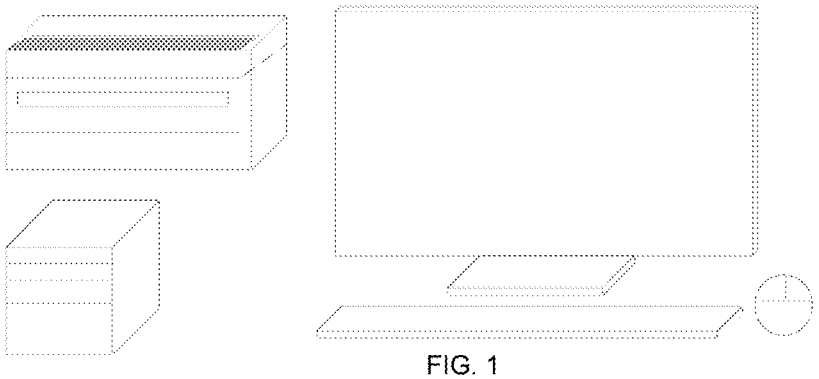


FIG. 1

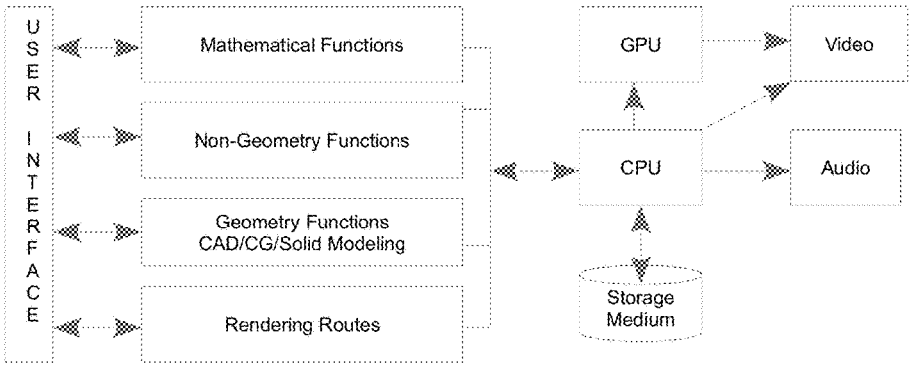


FIG. 2A

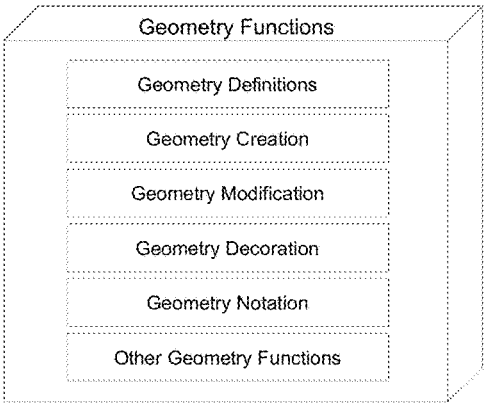


FIG. 2B

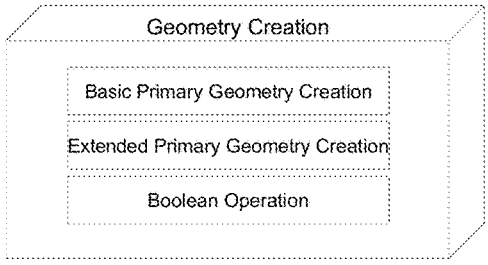


FIG. 2C

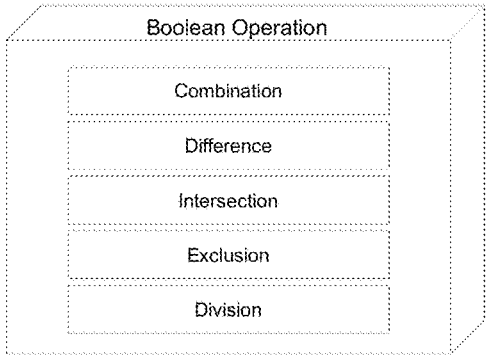


FIG. 2D

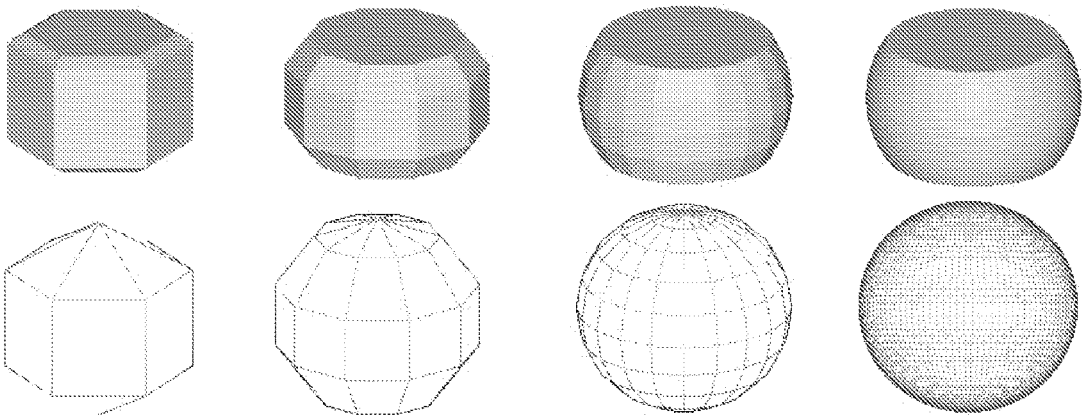


FIG. 3A Prior Art FIG. 3B Prior Art FIG. 3C Prior Art FIG. 3D Prior Art

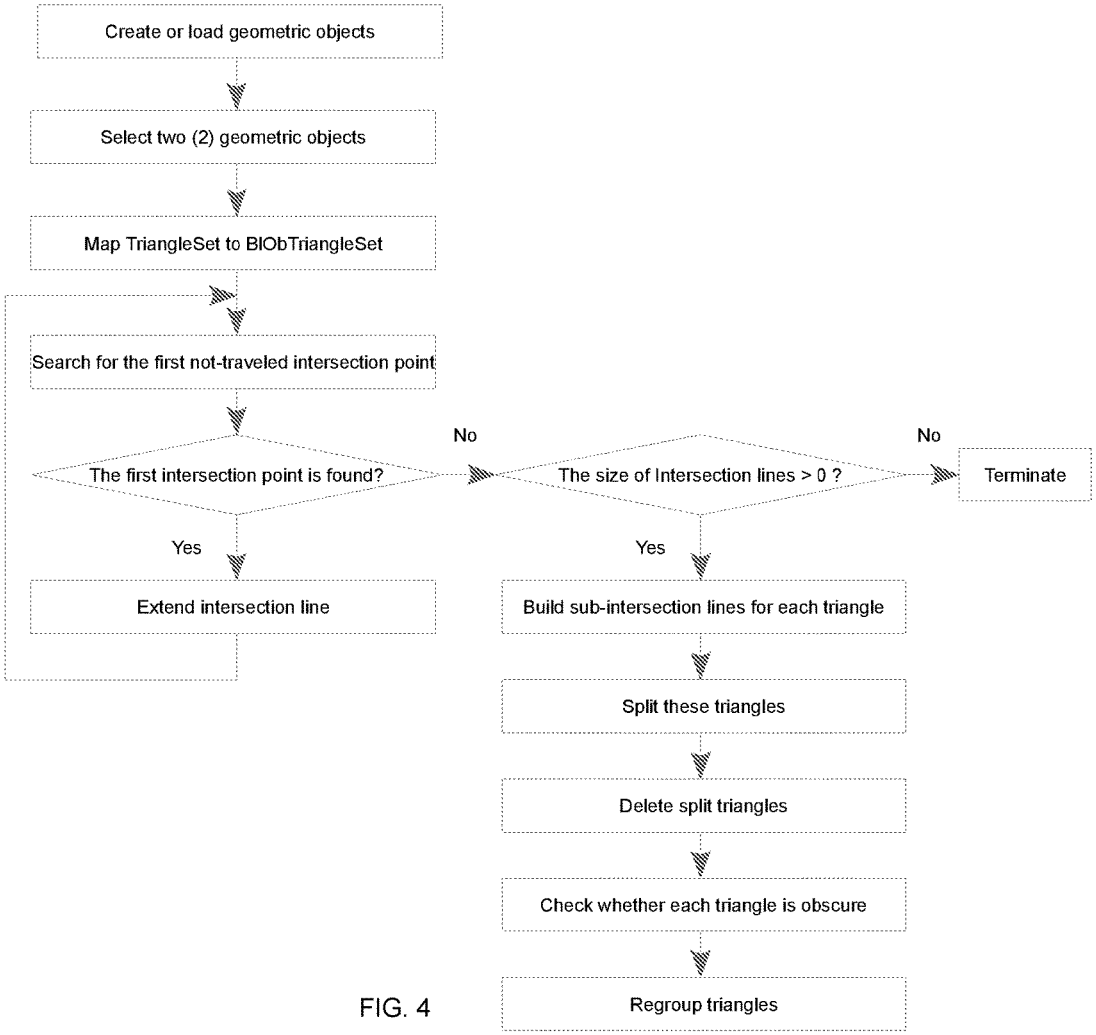


FIG. 4

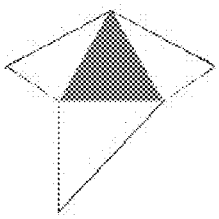


FIG. 5

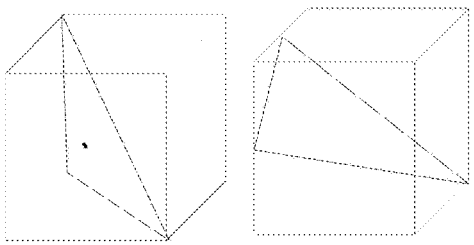


FIG. 6A

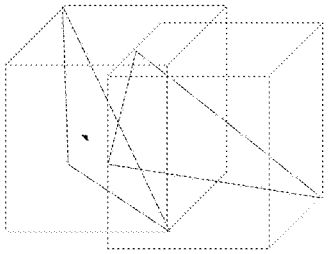


FIG. 6B

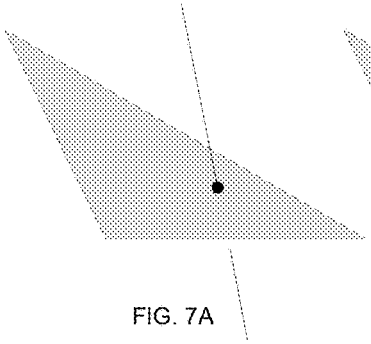


FIG. 7A

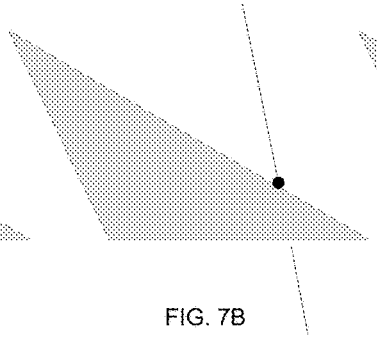


FIG. 7B

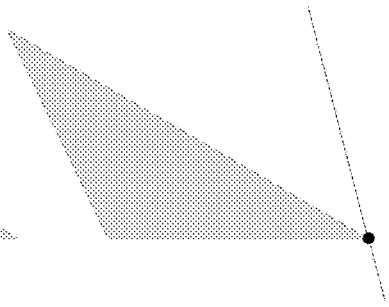


FIG. 7C

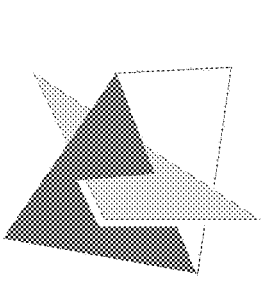


FIG. 8A

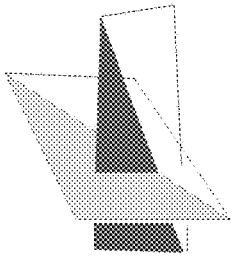


FIG. 8B

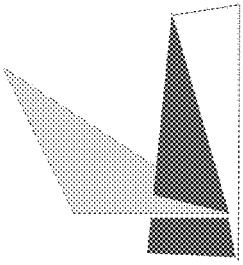


FIG. 8C

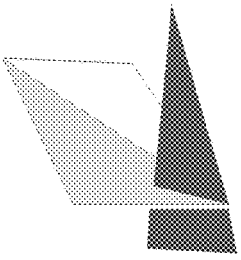


FIG. 8D

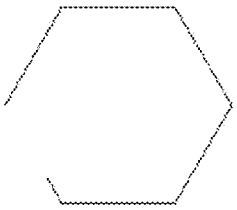


FIG. 9A

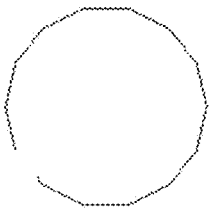


FIG. 9B

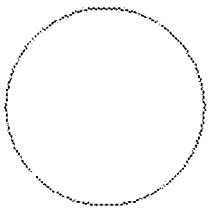


FIG. 9C

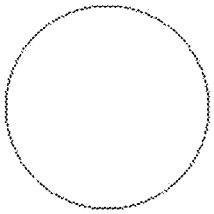


FIG. 9D

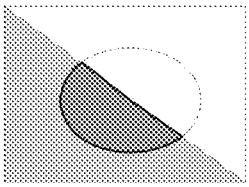


FIG. 10A

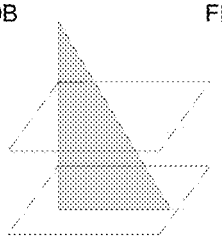


FIG. 10B

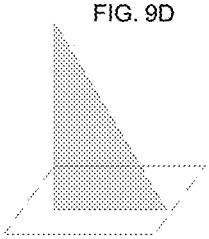


FIG. 10C

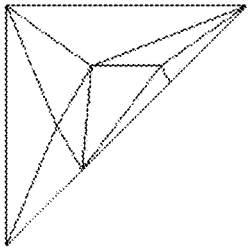


FIG. 11A

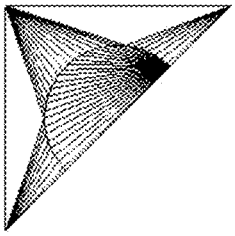


FIG. 11B

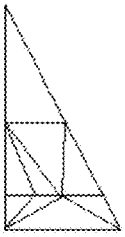


FIG. 11C

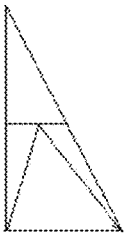


FIG. 11D

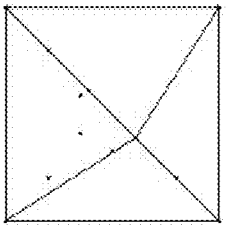


FIG. 12A

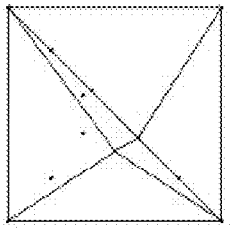


FIG. 12B

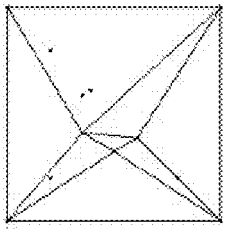


FIG. 12C

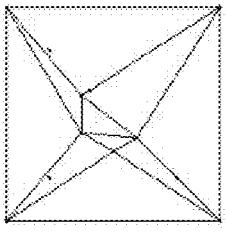


FIG. 12D

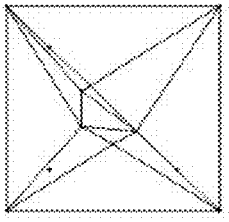


FIG. 12E

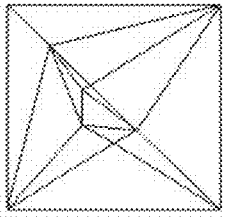


FIG. 12F

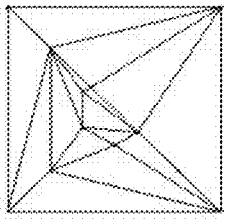


FIG. 12G

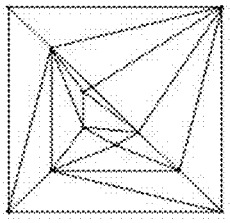


FIG. 12H

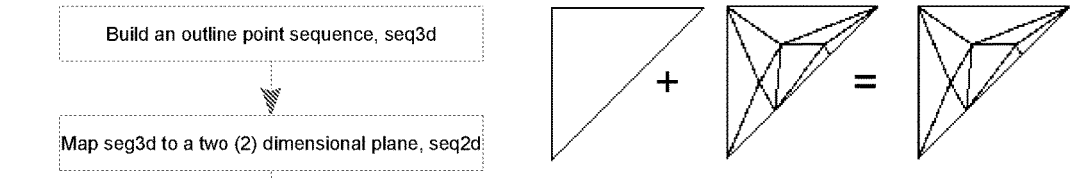


FIG. 14

FIG. 13

FIG. 15

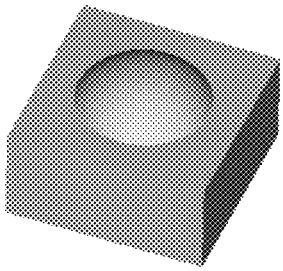


FIG. 16A

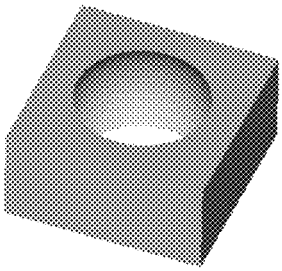


FIG. 16B

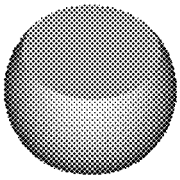


FIG. 16C

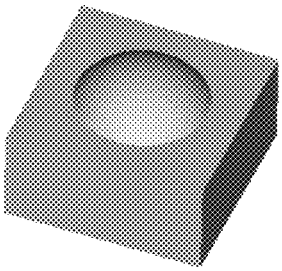


FIG. 16D

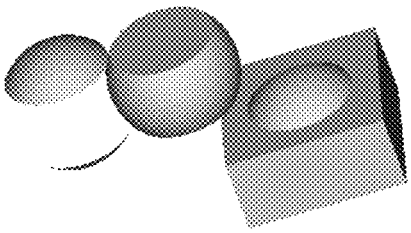


FIG. 16E

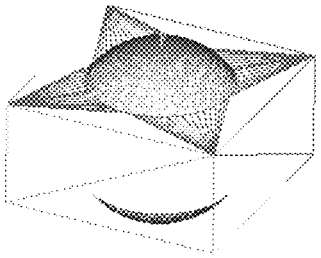


FIG. 16F

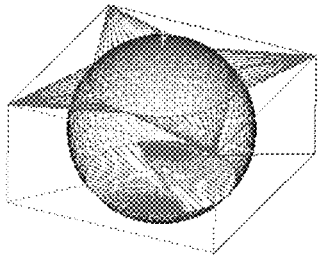


FIG. 16G

METHOD FOR IMMEDIATE BOOLEAN OPERATIONS USING GEOMETRIC FACETS

BACKGROUND

Field of the Invention

This invention provides an immediate Boolean operation method for building three (3) dimensional solid geometric models from primary geometric objects to Computer Aided Design, Computer Graphics, and Solid Modeling systems, which are widely used in product design, manufacturing, and simulation. Mechanic industry, culture and sports, everywhere there are geometric shapes, may have CAD/CG applications.

Related Art

Computer hardware is so highly developed that even an ordinary Personal Computer may be used to install and run a commercial CAD/CG system, which normally has Boolean operation functions including AND, OR, and NOT. PC components comprise input devices, such as a mouse and a keyboard, a main machine, a screen, and a printer. The software system contains geometric and non geometric functions. FIG. **1** shows the main PC components and FIGS. **2**A through **2**D depict a typical CAD/CG software system architecture.

Boolean operations provide a general process of building complex solid geometric objects from different geometric shapes, which include primary geometric objects, swept or extruded objects, to CAD/CG/Solid Modeling systems. Lee applied Boolean operations to divide surface [Lee U.S. Pat. No. 6,307,555].

Boolean operations may rely on Constructive Solid Geometry, CSG, to record primary geometry objects and operation sequence in a hierarchical way, which technically is easy to implement, whereas Boundary Representation, B-REP, is regarded as a more flexible way that supports more geometric object types like extended geometries [Gursoz, 1991].

This invention presents five (5) Boolean operation commands: combination, intersection, exclusion, difference, and division, which directly work on triangles decomposed from geometric facets used for rendering functions and do not require the data structure Constructive Solid Geometry or Boundary Representation. The data structures defined in this invention are a few of simple classes, the algorithms incorporated in this invention are concise and easy to implement, and the five (5) commands allow the user to create geometric models not only by selecting the types of geometric objects but also by defining their facets. FIGS. **3**A through **3**D present a box with **6** facets and a sphere with different facets make distinct results.

This invention presents different data structures and algorithms compared with CSG and B-REP, the algorithms include triangle-triangle intersections, building intersection lines, splitting each triangle with sub-intersection lines, and regrouping triangles to form Boolean operation results.

BRIEF SUMMARY OF THE INVENTION

This invention provides a set of data structures and algorithms for performing Boolean operations, which are used to build complex geometric models and work directly on triangles decomposed from geometric facets used as rendering data by computer hardware and rendering functions like OpenGL libraries. A geometric shape, for example, a sphere, a cone, a cylinder, a box, triangular facets, an extruded or swept object, is triangulated to form a set, noted as TriangleSet, for displaying. When two geometric objects are selected for performing a Boolean operation, neighboring triangles will be added to each triangle in TriangleSet to form another set, BlOpTriangleSet.

The second step of a Boolean operation this invention described is to search and build intersection lines between triangle sets. It starts with finding the first pair of intersecting triangles: this system builds an axis aligned minimum bounding box for each triangle and checks whether two bounding boxes overlap to decide if edge-triangle intersection needs to be calculated. Once the edge-triangle intersection point(s) falls inside a triangle, this system completes the searching task and stores the point data into an intersection line set.

To extend the current intersection line, this method traces neighboring triangles and calculates edge-triangle intersection points until the intersection line becomes closed or all triangles are traversed.

The third step of a Boolean operation this invention described is to split triangles. Each segment of the intersection lines references two (2) triangles, each of the triangles has at least one sub-intersection line that contains one or more segments, which divide a triangle into three (3) or more smaller triangles. After splitting the triangles, the original triangles are removed, and those smaller triangles are added to the BlOpTriangleSet.

The fourth step of a Boolean operation this invention described is to decide if each triangle is obscure or visible. If a triangle is enclosed by other triangles, it is obscure. A triangle is visible means it is outside another object.

The fifth step of a Boolean operation this invention described is to regroup the triangles: some of them have to be removed and some need to be put together, and there are five (5) cases for regrouping.

The final step of a Boolean operation this invention described is to map BlOpTriangleSet to TriangleSet.

BRIEF DESCRIPTION OF THE DRAWINGS

FIG. **1** shows the main personal computer components, which generally contain a main machine, input devices including a mouse and a keyboard, a display, and a printer. A highly developed CAD/CG system can run on a PC machine.

FIGS. **2**A through **2**D describe a software architecture in which a CAD/CG/Solid Modeling system uses Boolean operations to build geometric models.

FIGS. **3**A through **3**D represent that different facets make various results even their original geometric object types and sizes are the same: the left side example has less facets and the right side has more facets. In these examples, Boolean Intersection operations work on a box and a sphere.

FIG. **4** is a flowchart for immediate Boolean operations using geometric facets.

FIG. **5** depicts that a triangle has three (3) neighbors. Given a triangle and its two vertices, there is one and only one neighboring triangle in solid models.

FIGS. **6**A and **6**B show two minimum bounding boxes do not overlap and two boxes overlap each other. Each triangle virtually has a minimum bounding box. If two boxes do not overlap, the triangles contained in the two boxes do not intersect. If the boxes overlap, edge-triangle intersection calculation is required.

FIGS. **7**A through **7**C depict three (3) edge-triangle intersection cases: an intersection point falls inside a triangle, an intersection point locates on an edge of a triangle, an intersection point is a vertex of a triangle.

FIGS. **8**A through **8**D show the searching candidate set, which allows the system to travel next triangle for extending intersection lines by conducting edge-triangle calculation. Triangles filled with colors are the last pair triangles that intersect each other, the triangles not filled are referenced by the member m_NeigTri of the data structure Triangle3dEx, which guides the system searching a minimum set of triangles when building intersection lines. The set contains one triangle, two triangles, or zero.

FIGS. **9**A through **9**D show four (4) examples of intersection lines. A box intersects a sphere, which has different facet numbers.

FIGS. **10**A through **10**C give three (3) examples of sub-intersection lines in darker color. FIG. **10**A has one (1) sub-intersection line, **10**B two (2), and **10**C one (1).

FIGS. **11**A through **11**D show four (4) examples that sub-intersection lines divided a triangle into a set of triangles.

FIGS. **12**A through **12**H show a Delaunay mesh sequence in which each intersection point is inserted into the mesh step by step.

FIG. **13** is the flowchart of Delaunay mesh modified Watson method that created the sequence of FIGS. **12**A through **12**H.

FIG. **14** shows that a triangle and its Delaunay mesh. The original triangle is removed and only the Delaunay mesh is reserved for late computations.

FIG. **15** shows t-Buffer where t may be negative and positive. If the size of negative t and positive t is balanced in t-Buffer, the triangle concerned is closed by another object and is obscure.

FIGS. **16**A through **16**E show five (5) examples of Boolean operations conducted with a box and a sphere. FIGS. **16**F and **16**G depict the internal mesh of two Boolean operation resultants: combination and exclusion.

DETAILED DESCRIPTION

This invention defines these data structures: Point3dEx, Triangle3dEx, and BlOpTriangle3dSet that inherit Point3d, Triangle 3d, Triangle3dSet storing facets for rendering geometric objects. When performing a Boolean operation, the system maps rendering facets to BlOpTriangle3dSet and all following processes focus on the members and attributes of BlOpTriangle3dSet. FIG. **4** is the flowchart describing the main procedure of Boolean operations conducted by the present invention. After a Boolean operation completed, the system maps the resultant stored in BlOpTriangle3dSet to rendering facets.

Geometric Facets for Rendering

CAD systems render facets to represent a geometric object, such as a sphere, a cone, a box, a cylinder, an extruded or swept object. A facet may compose three (3) or more points, and facets are usually decomposed into triangles for easy calculations. A box has six (6) facets decomposed into twelve (12) triangles. A sphere may have eighteen (18) facets, composing twenty four (24) triangles. A sphere may also be rendered using more than one thousand (1,000) facets and triangles. FIGS. **3**A through **3**D show a sphere rendered with different facets. This method uses Triangle3dSet to note triangle set data structure for rendering a geometric object, it contains two (2) attributes: a three (3) dimensional point set and a triangle set, where Triangle3d references Point3d.

```
class Triangle3dSet
{
DataSet<Point3d> m_PointSet;
DataSet<Triangle3d> m_Tri angleSet;
};
class Triangle3d
{
Reference<Point3d> m_P0, m_P1, m_P2;
};
class Point3d
{
DataTypeI m_X, m_Y, m_Z;
};
```

Triangles for Boolean Operations

The Boolean Operation method described in this invention defined three (3) key classes: BlOpTriangleSet, Triangle3dEx, and Point3dEx.

```
class BlOpTriangleSet
{
  DataSet<Point3dEx>          m_PointSet;
  DataSet<Triangle3dEx>       m_TriangleSet;
};
class Point3dEx : Point3d
{
    DataTypeII      m_ID; // position and sequence index
    DataTypeIII     m_X, m_Y, m_Z; // DataType III
                    may be different from DataTypeI
};
class Triangle3dEx : Triangle3d
{
    DataTypeII      m_ID;
    Plane           m_Plane;
    DataTypeIV      m_Normal[3];
    Triangle3dEx*m_NeigTri[3]; // neighboring triangles
};
```

DataTypeII may be int, long, unsigned long, or other integer types. DataTypeIII is a floating point data type, such as float, double, even long double.

The class Triangle3dEx specifies each triangle may have 3 neighboring triangles, and every triangle is stored just one (1) copy in BlOpTriangleSet. Given the box example, the simplest way it has 12 triangles, even each of them has three (3) neighbors, BlOpTriangleSet still stores a total of 12 triangles.

Technically Triangle3d may have the attribute m_Normal. If DataTypeI and DataTypeIV are the same type, for example, double, the attribute m_Normal can be inherited.

Data Mapping

The process of mapping Triangle3dSet to BlOpTriangleSet copies point set and triangle set from rendering statue and fills default attributes. Data mapping contains the following procedures:

1) Copy points from Triangle3dSet to BlOpTriangleSet and ensure there are not identical points.

2) Copy triangles from Triangle3dSet to BlOpTriangleSet.

3) For each triangle in BlOpTriangleSet, set its neighboring triangles.

4) Calculate the normal and build the plane equation for each triangle in BlOpTriangleSet.

Remark 1: Given two (2) points a and b, if $|x_a-x_b|<\varepsilon$ and $|y_a-y_b|<\varepsilon$ and $|z_a-z_b<\varepsilon$, where ε is a positive float pointing number, for example 5.0e-16, then b is identical to a.

Remark 2: When mapping points from rendering data to BlOpTriangleSet, the system checks if there is an identical point in BlOpTriangleSet.

Remark 3: A triangle, which has three (3) points, defines a plane whose mathematical formula is ax+by+cz+d=0 and the class Plane internally records it as an array of four (4) numbers, such as double m_ABCD[4].

Remark 4: A triangle, if its three (3) points are not identical, always has a valid normal. Even it is related to m_ABCD, a separate copy makes things more clear and easy to handle later.

Remark 5: Every triangle has three (3) edges, when there are no duplicated points, it has three (3) neighboring triangles in solid models. FIG. **5** shows an example: a triangle filled with dark color and its three (3) neighbors.

The First Intersection Point

Every triangle has three (3) vertices, which define a minimum bounding box. This method adopted the concept of axis aligned minimum bounding box.

Given a pair of triangles, if their bounding boxes do not overlap, the two triangles have no intersection point; otherwise, this method carries out edge-triangle intersection calculations.

If an edge of a triangle T_a intersects with a plane defined by a triangle T_b and the intersection point pet falls inside T_b, then pet is the first intersection point. If pet is outside of T_b, then switch the triangle position in the pair, (T_a, T_b) changed to (T_b, T_a), and conduct edge-triangle intersection calculations.

Given the i-th edge of a triangle T_a, i∈[0, 2], its formula is: $p=p_i+t*(p_{(i+1)\%3}-p_i)$, and the plane defined by the triangle T_b, its formula is: ax+by+cz+d=0. Hereinafter the symbol % expresses the modulo operator. If the two formulas have a solution, the edge intersects with the plane. If the edge-plane intersection point falls inside the triangle T_b, then the point is the edge-triangle intersection point.

Extending an Intersection Line

This method defines a data structure for recording an intersection point as PntEgTri:

```
class PntEgTri
{
    Triangle3dEx    *m_Tri0, *m_Tri1;
    DataTypeII      m_EdgeIndex;
    DataTypeII      m_PointPosi;
    Point3dEx       m_Point;
    Point3dEx       *m_PntGlobalIndexA, *m_PntGlobalIndexB;
};
```

According to the location of an intersection point on a triangle, a PntEgTri, simply said pet, can be classified into three (3) categories shown in FIGS. **7**A through **7**C.

1) The most popular case is edge-triangle intersection, pet locates on an edge of triangle T_a and inside triangle T_b.
2) Edge-edge intersection, pet locates on an edge of triangle T_a and on an edge of triangle T_b.
3) Edge-vertex intersection, pet locates on an edge of triangle T_a and on a vertex of triangle T_b.

To extend an intersection line, this method catches next neighboring triangle(s) and checks edge-triangle intersection until the intersection line gets closed or all triangles are traversed.

Sub-Intersection Line

An intersection line passes through a set of triangles and divides each triangle into multi partitions. The segments of an intersection line inside a triangle make up a sub-intersection line. FIGS. **10**A through **10**C show three examples in which the dark lines are sub-intersection lines. In practice, a triangle may have zero (0), one (1), two (2), or three (3) sub-intersection lines.

The following algorithm shows how to get a valid reference to a triangle that has at least one sub-intersection line:

```
for each intersection line
for each intersection point, get the triangle references:
    (m_Tri0, m_Tri1)
    for each triangle of the triangle pair, if it is not split
        for each intersection line
            search and build a sub-intersection line
```

Given a valid triangle and an intersection line, to decide if a pet belongs to the sub-intersection line of the triangle, this method checks whether

1) pet is on an edge of the triangle,
2) or pet is inside the triangle,
3) or pet equals a vertex of the triangle.

Splitting a Triangle

Given a set of sub-intersection lines, to split a triangle, this method

1) Removes duplicated pets. If neighboring pets are identical, this method reserves just one copy.
2) Identifies the position of end pets: checks each pet locates on which edge of the triangle.
3) Splits the upper partition, lower partition, and middle partition of the triangle where applicable.

Given a set of points on a plane that represents a partition of a triangle, to decompose the plane into a group of triangles, this invention modified Delaunay 2D mesh Watson method, which is published in 1981 [Watson, 1981].

A Delaunay 2D mesh has three (3) data set: triangle set that holds the generated triangles, deleted triangle set that stores just deleted triangles, and polygon that records the outline of deleted triangle set.

The modified Delaunay 2D mesh method contains the following steps:

1) Build an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable;
2) Map the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane;
3) Add four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points;
4) Assume that one dialog line of the bounding box splits the box into two (2) triangles and add them into the triangle set;
5) Insert every point except that bounding ones into the triangle set.
   a) For each point, check every triangle in the triangle set whether its circumcircle contains the point or the last segment of the outline passes through the triangle. If the condition is true, erase it from the triangle set and add it to the deleted triangle set.

b) Use the deleted triangle set to extend polygon and clear the deleted triangle set immediately.

c) Use the polygon to generate triangles and add them to the triangle set.

6) Delete boundary triangles from the triangle set.

7) Map the triangle set to three (3) dimensional triangles and add them to BlOpTriangleSet.

FIGS. **12**A through **12**H show a Delaunay 2D mesh sequence.

Deleting Split Triangles

In the above step, a split triangle got a mark. After all triangles have been traversed, this method deletes the marked triangles. FIG. **14** shows a deletion result.

Obscure Facets

Given two sets of triangles A and B, if A bounds a triangle of B, T_b, then T_b is obscure; if B bounds a triangle of A, T_a, then T_a is obscure.

To check whether a triangle T is bounded by an object O, this invention uses the following steps.

(1) Calculate the centred, c, of the triangle, T.

(2) Build a line I:p=c+t*N, which passes through the centroid and along the normal N of the triangle T.

(3) For each triangle T_0 of the object O, calculate line-plane intersection point. If there is a valid intersection point pet that falls inside the triangle T_0, then calculate t that is determined by centroid c and the pet, and add t to a depth buffer, buffer T.

(4) Check the size of negative t and positive t stored in buffer T. If the two sizes are equal, then the triangle T is bounded and obscure.

Regrouping the Facets

This invention presents five (5) kinds of Boolean operations: combination, intersection, exclusion, difference, and division, each of them has a different regrouping procedure. The combination operation, logically it is OR, combines two solid geometric objects and generates a new object, which normally discards obscure partitions and reserves visible ones viewing from outside, has the following procedure.

1) Delete obscure triangles of object A;

2) Delete obscure triangles of object B;

3) Merge the triangles of object A and B.

The intersection operation, logically it is AND, which creates a solid geometric object using public section(s) of two geometric objects and discards any partitions of A and B outside the shared public section(s), has the following procedure.

1) Delete NOT obscure triangles of object A;

2) Delete NOT obscure triangles of object B;

3) Merge the triangles of object A and B.

The exclusion operation, which builds a solid geometric object by removing public section(s) of two geometric objects and keeps not shared partitions, has the following procedure.

1) Copy object A's obscure triangles to a buffer, buffer A;

2) Delete obscure triangles from object A;

3) Copy object B's obscure triangles to object A;

4) Delete obscure triangles from object B;

5) Copy the triangles in buffer A to object B;

6) Reverse the normal of every obscure triangle of A and B;

7) Merge the triangles of the two objects.

The difference operation, which cuts geometric object A with another object B by removing any partitions of A inside B, has the following procedure.

1) Delete obscure triangles of object A;

2) Delete NOT obscure triangles of object B;

3) Reverse the normal of every triangle of object B;

4) Merge triangles of object A and B.

The division operation, which divides two solid geometric object A and B into three (3) objects, public section(s) of the two geometric objects, the NOT shared partitions of A and partitions of B, has the following procedure.

1) Copy object A's obscure triangles to a buffer, buffer A;

2) Copy object B's obscure triangles to buffer A;

3) Copy object A's obscure triangles to another buffer, buffer B;

4) Delete object A's obscure triangles;

5) Copy object B's obscure triangles to object A;

6) Delete object B's obscure triangles;

7) Copy object A's obscure triangles stored in buffer A to object B;

8) Reverse the normal of every obscure triangles of A and B.

Mapping to Rendering Facets

Once a Boolean operation is finished, this method maps BlOpTriangleSet to rendering triangles.

1) Each Point3dEx of BlOpTriangleSet is mapped to a Point3d of TriangleSet;

2) Each Triangle3dEx of BlOpTriangleSet is mapped to a Triangle3d of TriangleSet.

U.S. PATENT DOCUMENTS

U.S. Pat. No. 6,307,555, 10-2001, Lee; Eugene T. Y. 345/421.

OTHER PUBLICATIONS

"Boolean Set Operations on Non-Manifold bounding Representation Objects", E. Gursoz et al., Computer-Aided Design 23 (1991) January/February No. 1 London, GB.

"Computing the n-dimensional Delaunay tessellation with application to Voronoi polytopest", D. F. Watson, The Computer Journal 24 (2) 1981.

What is claimed:

1. A method that performs immediate Boolean operations using geometric facets of geometric objects implemented in a computer system and operating with a computer, the method comprising:

mapping rendering facets to extended triangles that contain neighbors;

building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection line by detecting whether two minimum bounding boxes overlap and performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection point of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed;

splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end inter-

section points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion;

checking each triangle whether it is obscure or visible for Boolean operations;

regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets; and

mapping extended triangles to rendering facets.

2. The method of claim **1** wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets without the data structure Constructive Solid Geometry and Boundary Representation.

3. The method of claim **1** wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets with the data structure CSG or B-REP.

4. The method of claim **1** wherein searching for the first pair of triangles that hold a start point of an intersection line and searching neighboring triangles of the last triangle pair that hold the last intersection point composed the procedure for building an intersection line that usually repeats more than one time when building intersection lines use the minimum bounding boxes to detect whether two triangles do not overlap and carry out edge-triangle intersection calculations comprising the steps ensuring that the intersection points are exact and the intersection lines are not approximate curves: building the formula $p=p_i+t*(p_{(i+1)\%3}-p_i)$ for expressing the i-th edge of the triangle T_a that is one triangle inside the triangle pair, building the formula $ax+by+cz+d=0$ for recording the plane defined by the triangle T_b that is another triangle inside the triangle pair, and getting the solution of the two linear formulas.

5. The method of claim **1** wherein searching for the first pair of triangles and searching neighboring triangles calculate edge-triangle intersection and employee neighboring triangles ensuring that direct calculation of edge-edge intersection is replaced by verifying whether a point is on an edge of a triangle.

6. The method of claim **1** wherein splitting each triangle projects the three (3) dimensional triangle and all its sub-intersection lines onto a two (2) dimensional plane and builds Delaunay 2D mesh with modified Watson method that defines a triangle set, a deleted triangle set, and a polygon, dividing the triangle into different partitions even when the sub-intersection lines are not convex, comprising of: building an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable; mapping the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane defined by the triangle; adding four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points; assuming that one dialog line of the bounding box splits the box into two (2) triangles and adding them into the triangle set; inserting every point except bounding ones into the triangle set with the steps: for each point, checking every triangle in the triangle set whether its circumcircle contains the point or the last segment of the outline passes through the triangle, and when the condition is met, erasing it from the triangle set and adding it to the deleted triangle set, using the deleted triangle set to extend the polygon and clearing the deleted triangle set immediately, and using the polygon to generate triangles and adding them to the triangle set; deleting boundary triangles from the triangle set; and mapping the triangle set to three dimensional triangles and adding them to BlOpTriangleSet.

7. The method of claim **1** wherein triangles are classified as either visible, in which a visible triangle is not enclosed by a geometric object, or obscure in which an obscure triangle is enclosed by a geometric object.

8. The method of claim **1** wherein checking each triangle whether it is obscure or visible utilizes t-Buffer further comprising:

calculating the centroid c of triangle T_a that belongs to geometric object A;

building a line I: p=c+t*N passing through the centroid c and along the normal of T_a;

for each triangle T_b of object B, checking whether I intersects with T_b at an interior point and adding t to a depth buffer, t-Buffer, and setting T to be obscure when the size of negative t equals to that of positive tin t-Buffer.

9. The method of claim **1** wherein a Boolean operation that is a combination, an intersection, an exclusion, a difference, or a division, regroups facets for constructing its operational result using one or more steps of: deleting obscure or visible triangles of an object, copying obscure triangles of an object to a buffer or copying triangles from a buffer to an object, reversing the normal of each triangle of an object, and merging the triangles of the objects to form new extended triangle sets.

10. The method of claim **1** wherein the extended triangles are directly mapped to rendering facets for being displayed and providing data to next Boolean operations.

11. A computer system consisting of hardware and software that performs immediate Boolean operations using rendering facets of geometric objects, the system comprising:

a computer with input devices for entering data and commands, and a display device showing user interface, geometric objects, and additional data, having a medium storing geometric data and instructions that make up of a software system, or having a microchip or integrated circuit embedding partially or totally the instructions, and a processor that executes the steps of:

creating, modifying or loading primary geometric objects including swept and extruded ones and relocating them at different positions or orientations with input devices of the computer;

selecting two of the geometric objects;

mapping rendering facets to extended triangles that contain neighbors;

building intersection lines starting with and ending with searching for the first pair of triangles that hold a start point of an intersection line by detecting whether two minimum bounding boxes overlap and by performing edge-triangle intersection calculations for locating an intersection point, then searching neighboring triangles of the last triangle pair that holds the last intersection point to extend the intersection line until the first intersection point is identical to the last intersection

point of the intersection line ensuring that the intersection line gets closed or until all triangles are traversed;

splitting each triangle through which an intersection line passes using modified Watson method, wherein the modified Watson method includes removing duplicate intersection points, identifying positions of end intersection points, and splitting portion of each triangle including an upper portion, a lower portion, and a middle portion;

checking each triangle whether it is obscure or visible for Boolean operations;

regrouping facets in separate steps that includes copying triangles, deleting triangles, reversing the normal of each triangle of a geometric object, and merging reserved triangles to form one or more new extended triangle sets; and

mapping extended triangles to rendering facets.

12. The system of claim **11** wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets without the data structure Constructive Solid Geometry indicating the acronym CSG, and Boundary Representation indicating the acronym B-REP.

13. The system of claim **11** wherein any Boolean operations that use rendering facets of the geometric objects to create new geometric objects, including combination, intersection, exclusion, difference, and division, map rendering facets to extended triangles, build intersection lines, split each triangle through which an intersection line passes, check each triangle whether it is obscure or visible, regroup facets to form new extended triangle sets, and map extended triangles to rendering facets without with the data structure CSG or B-REP.

14. The system of claim **11** wherein searching for the first pair of triangles that hold a start point of an intersection line and searching neighboring triangles of the last triangle pair that hold the last intersection point composed the procedure for building an intersection line that usually repeats more than one time when building intersection lines use the minimum bounding boxes to detect whether two triangles do not overlap and carry out edge-triangle intersection calculations comprising the steps ensuring that the intersection points are exact and the intersection lines are not approximate curves: building the formula $p=p_i+t*(p_{(i+1)\%3}-p_i)$ for expressing the i-th edge of the triangle T_a that is one triangle inside the triangle pair, building the formula $ax+by+cz+d=0$ for recording the plane defined by the triangle T_b that is another triangle inside the triangle pair, and getting the solution of the two linear formulas.

15. The system of claim **11** wherein searching for the first pair of triangles and searching neighboring triangles calculate edge-triangle intersection and employee neighboring triangles ensuring that direct calculation of edge-edge intersection is replaced by verifying whether a point is on an edge of a triangle.

16. The system of claim **11** wherein splitting each triangle builds Delaunay 2D mesh with modified Watson method that defines a triangle set, a deleted triangle set, and a polygon, dividing the triangle into different partitions even when the sub-intersection lines are not convex, comprising of: building an outline point sequence that links sub-intersection lines and vertices of the triangle where applicable; mapping the three (3) dimensional point sequence to two (2) dimensional points according to the aspect of the plane defined by the triangle; adding four (4) points to form a bigger bounding box that encloses all the two (2) dimensional points; assuming that one dialog line of the bounding box splits the box into two (2) triangles and adding them into the triangle set; inserting every point except bounding ones into the triangle set with the steps: for each point, checking every triangle in the triangle set whether its circumcircle contains the point or the last segment of the outline passes through the triangle, and when the condition is met, erasing it from the triangle set and adding it to the deleted triangle set, using the deleted triangle set to extend the polygon and clearing the deleted triangle set immediately, using the polygon to generate triangles and adding them to the triangle set; deleting boundary triangles from the triangle set; mapping the triangle set to three dimensional triangles and adding them to BlOpTriangleSet.

17. The system of claim **11** wherein triangles are classified as either visible, in which a visible triangle is not enclosed by a geometric object, or obscure in which an obscure triangle is enclosed by a geometric object.

18. The system of claim **11** wherein checking each triangle whether it is obscure or visible utilizes t-Buffer further comprising:

calculating the centroid c of triangle T_a that belongs to geometric object A;

building a line I: p=c+t*N passing through the centroid c and along the normal of T_a;

for each triangle T_b of object B, checking whether I intersects with T_b at an interior point and adding t to a depth buffer, t-Buffer; and setting T to be obscure when the size of negative t equals to that of positive tin t-Buffer.

19. The system of claim **11** wherein a Boolean operation that is a combination, an intersection, an exclusion, a difference, or a division, regroups facets for constructing its operational result using one or more steps of: deleting obscure or visible triangles of an object, copying obscure triangles of an object to a buffer or copying triangles from a buffer to an object, reversing the normal of each triangle of an object, and merging the triangles of the objects.

20. The system of claim **11** wherein the extended triangles are directly mapped to rendering facets for being displayed and providing data to next Boolean operations.

* * * * *