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Rouse

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(54) **WIRELESS POWER TRANSFER SYSTEMS, METHODS AND TRANSMITTERS THEREFOR**

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H01P 3/16 (2006.01)

(52) **U.S. Cl.**
CPC ***H02J 50/12*** (2016.02); ***H01P 3/16*** (2013.01)

(58) **Field of Classification Search**
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USPC 307/104, 9.1, 11, 75; 257/414; 438/48; 398/116
See application file for complete search history.

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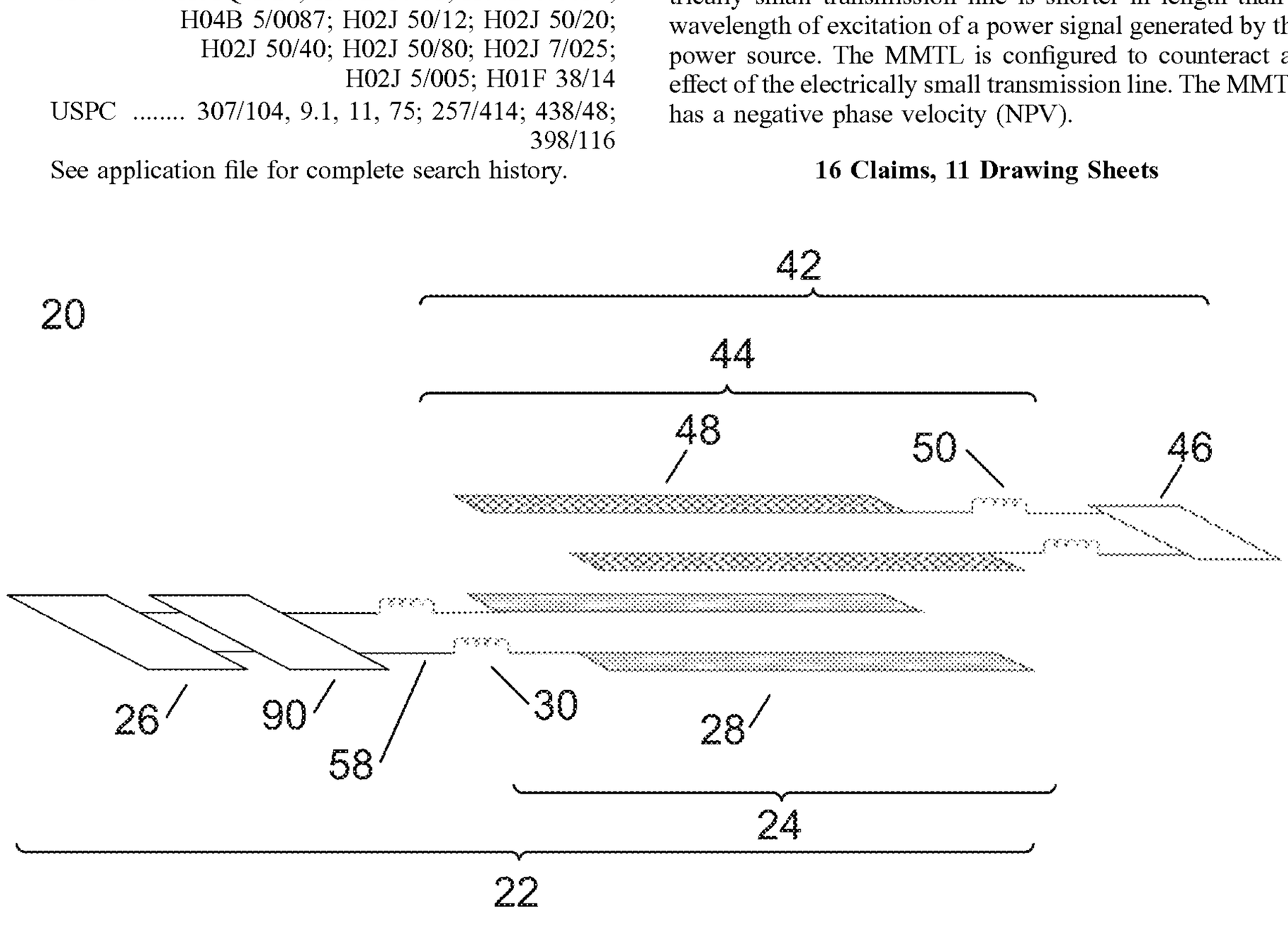
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(57) **ABSTRACT**

A transmitter comprises a power source; a transmit resonator electrically connected to the power source via an electrically small transmission line; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line. The transmit resonator comprises at least two electrodes; and at least one inductive coil electrically connected to the electrodes. The transmit resonator is configured to generate an electric field when excited by the power source. The electrically small transmission line is shorter in length than a wavelength of excitation of a power signal generated by the power source. The MMTL is configured to counteract an effect of the electrically small transmission line. The MMTL has a negative phase velocity (NPV).

16 Claims, 11 Drawing Sheets



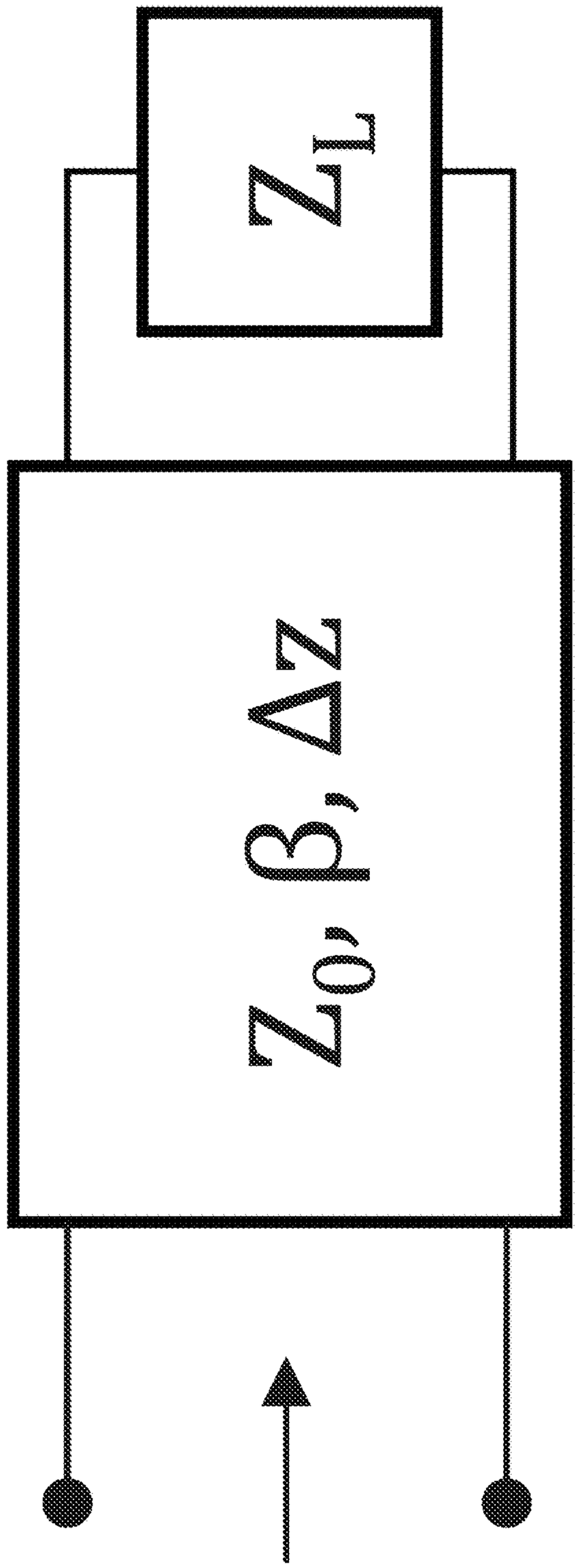


Figure 1
(Prior Art)

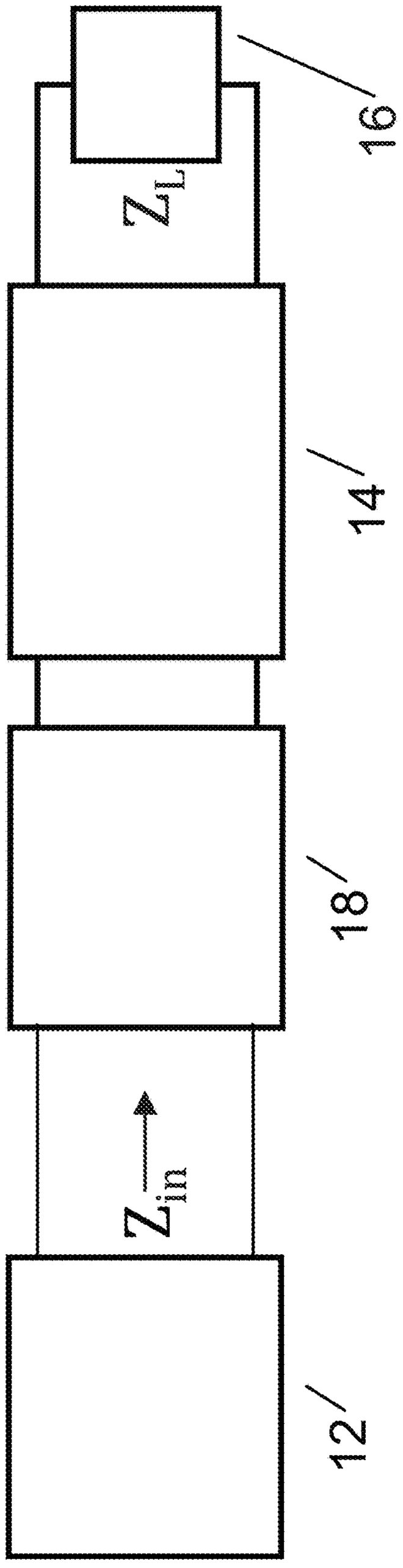


Figure 2

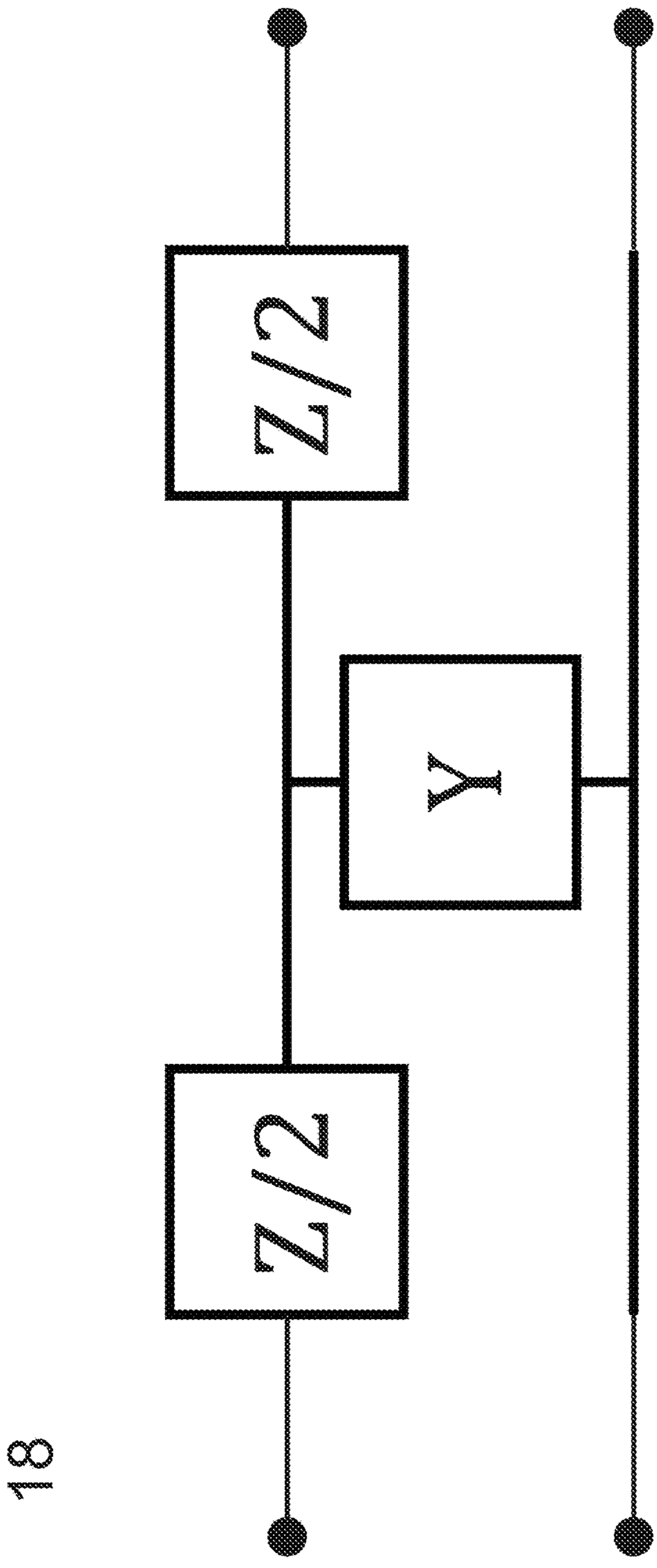


Figure 3

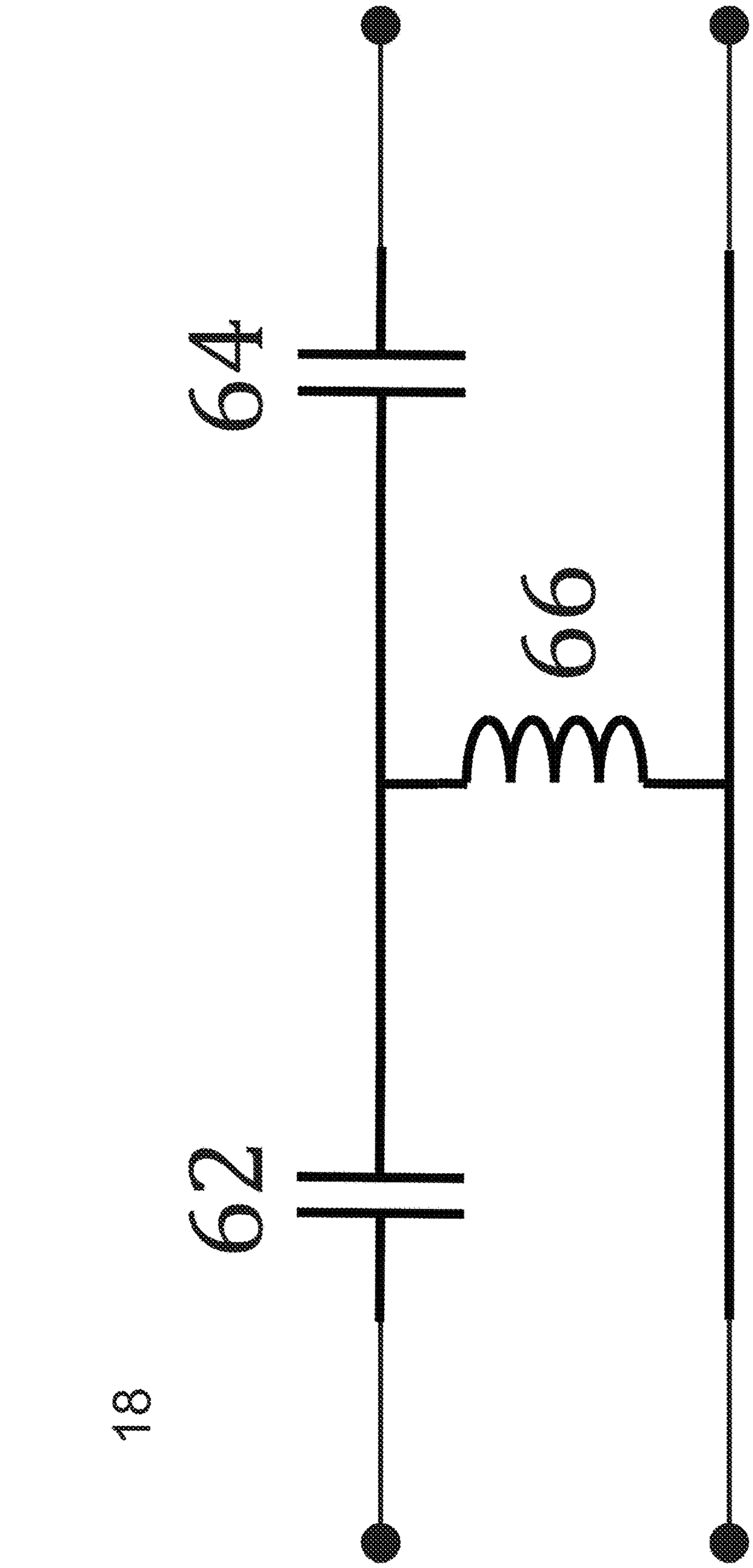


Figure 4

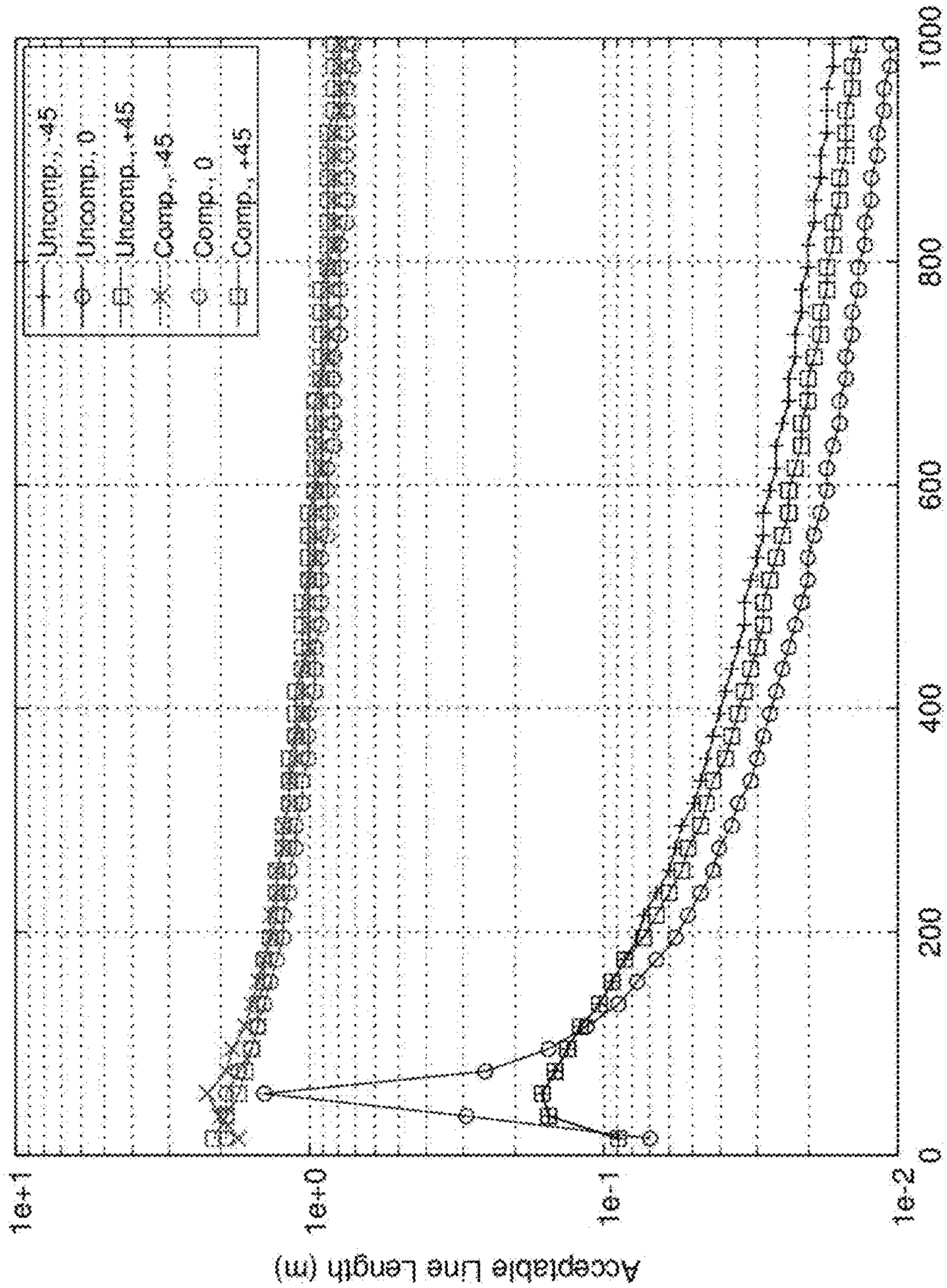

1000
800
600
400
200
0
Load Impedance Magnitude (ohms)
1e+1
1e+0
1e-1
1e-2
Acceptable Line Length (m)
Uncomp., -45
Uncomp., 0
Uncomp., +45
Comp., -45
Comp., 0
Comp., +45

Figure 5

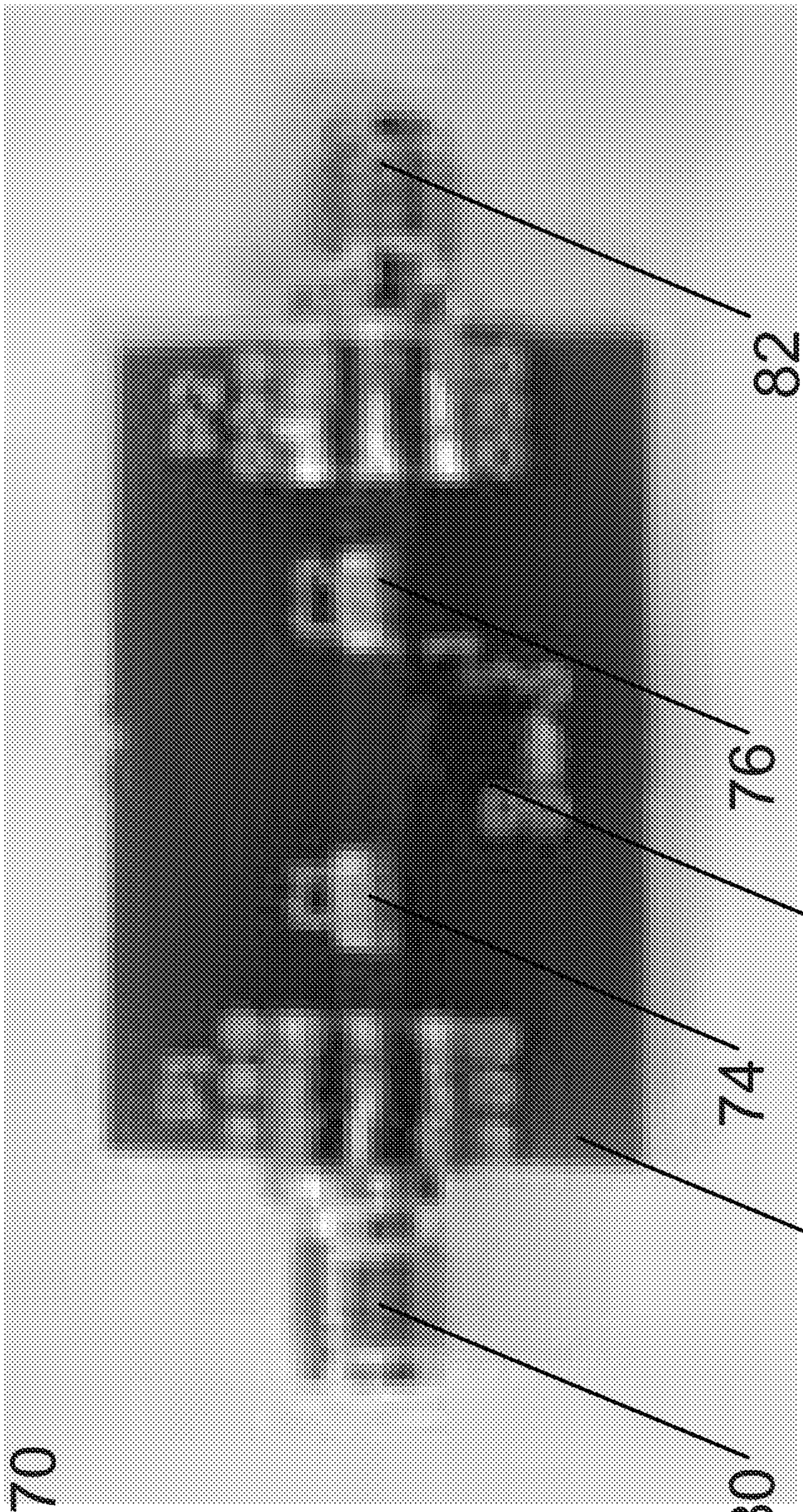


Figure 6

Figure 7a

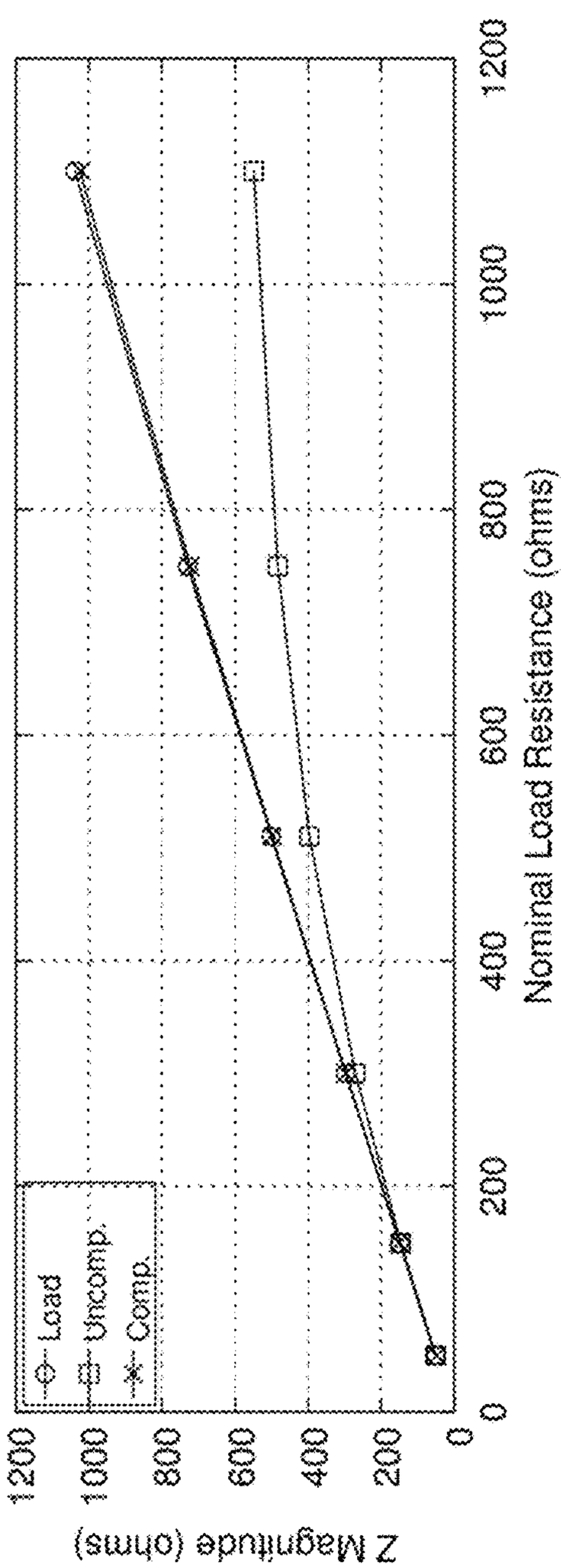


Figure 7b

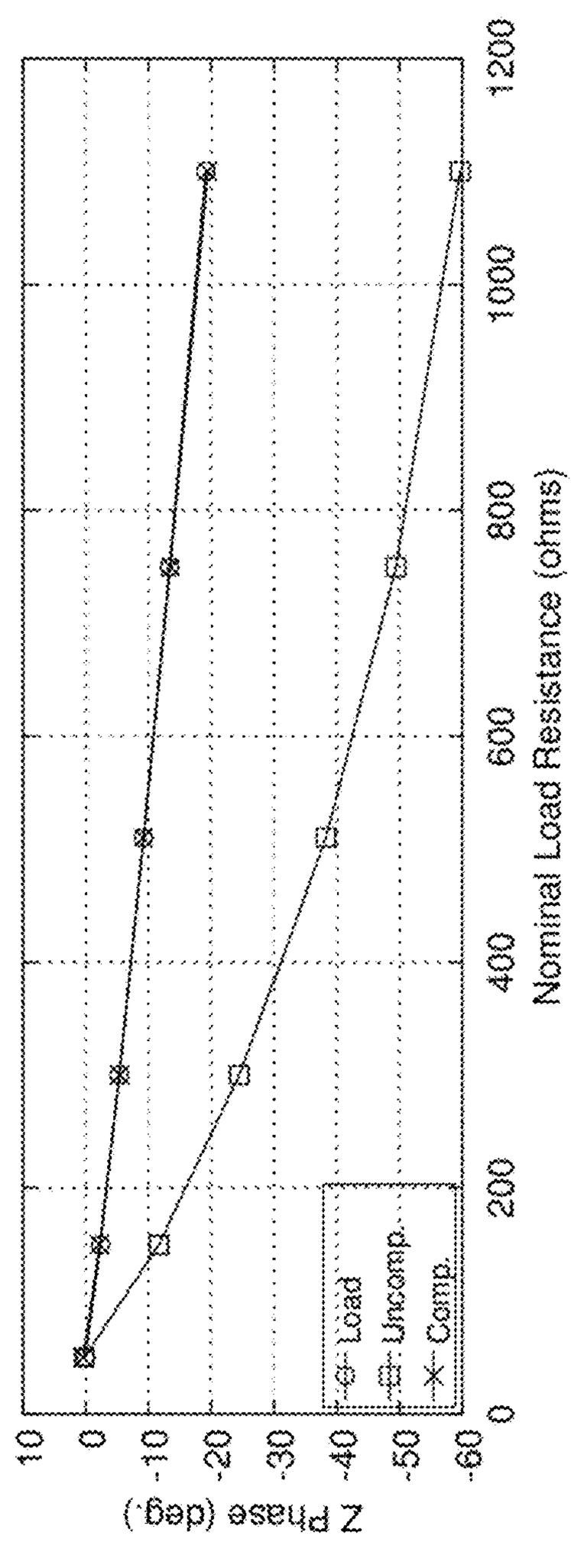


Figure 8a

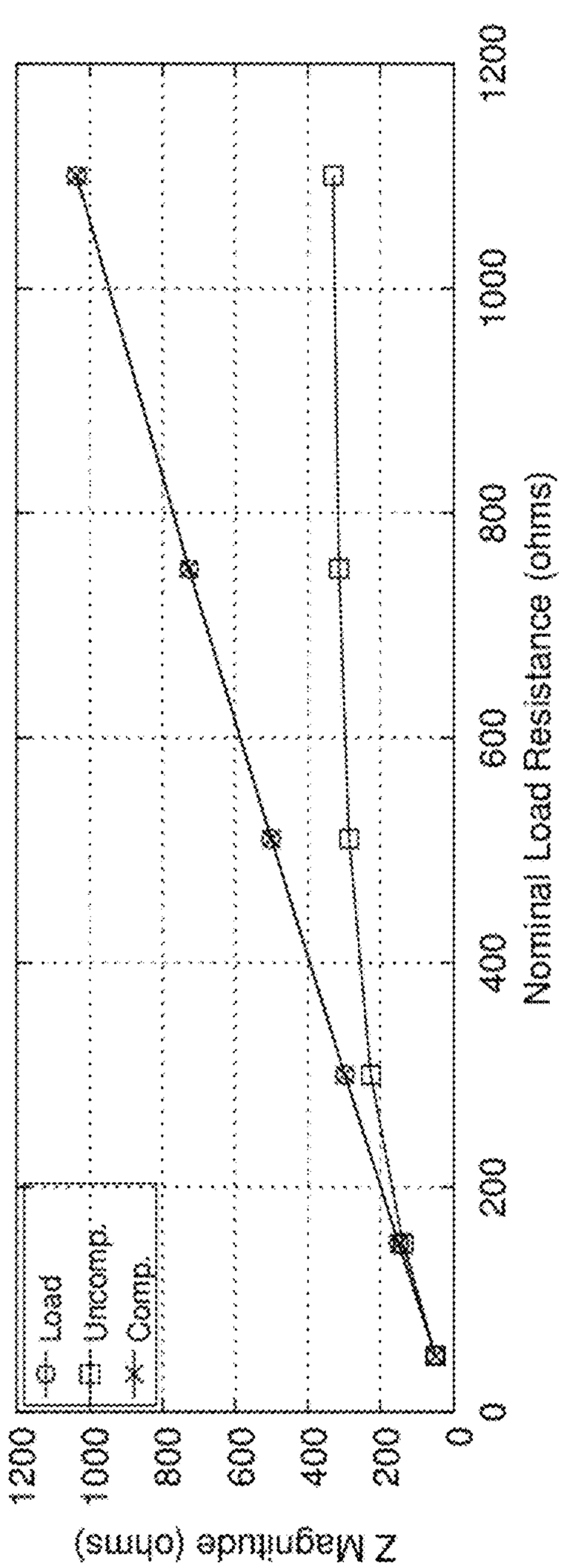


Figure 8b

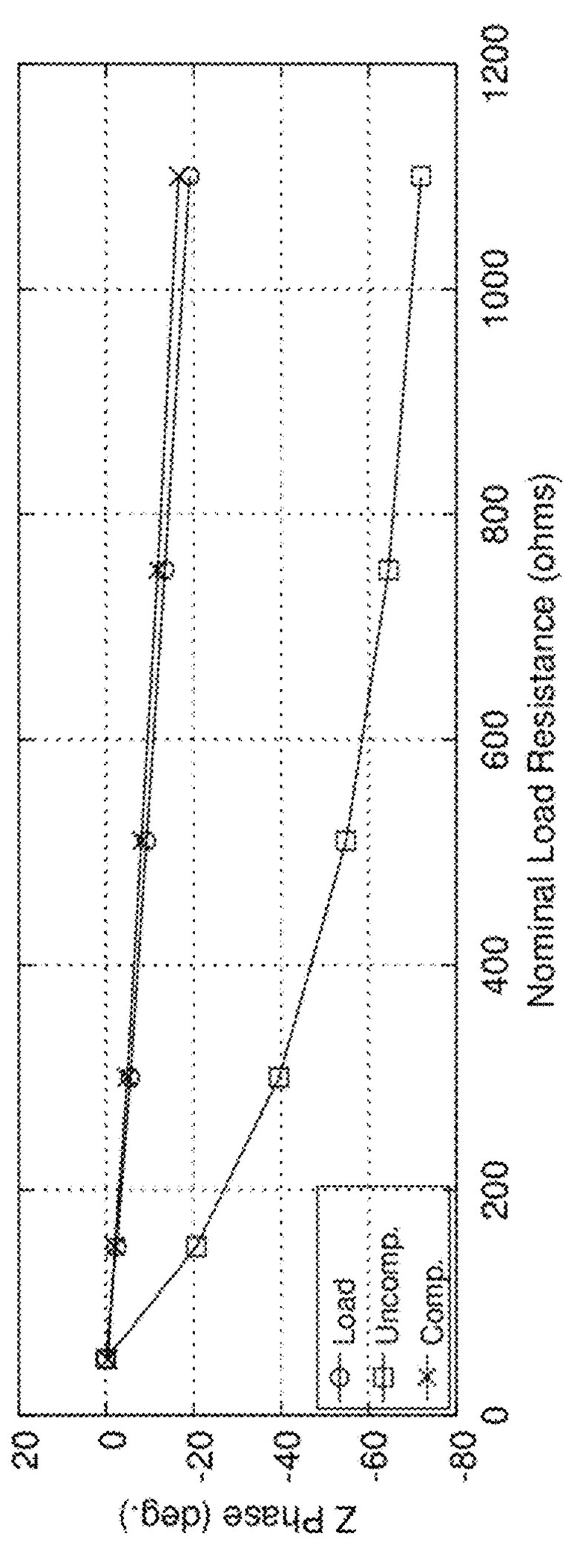


Figure 9a

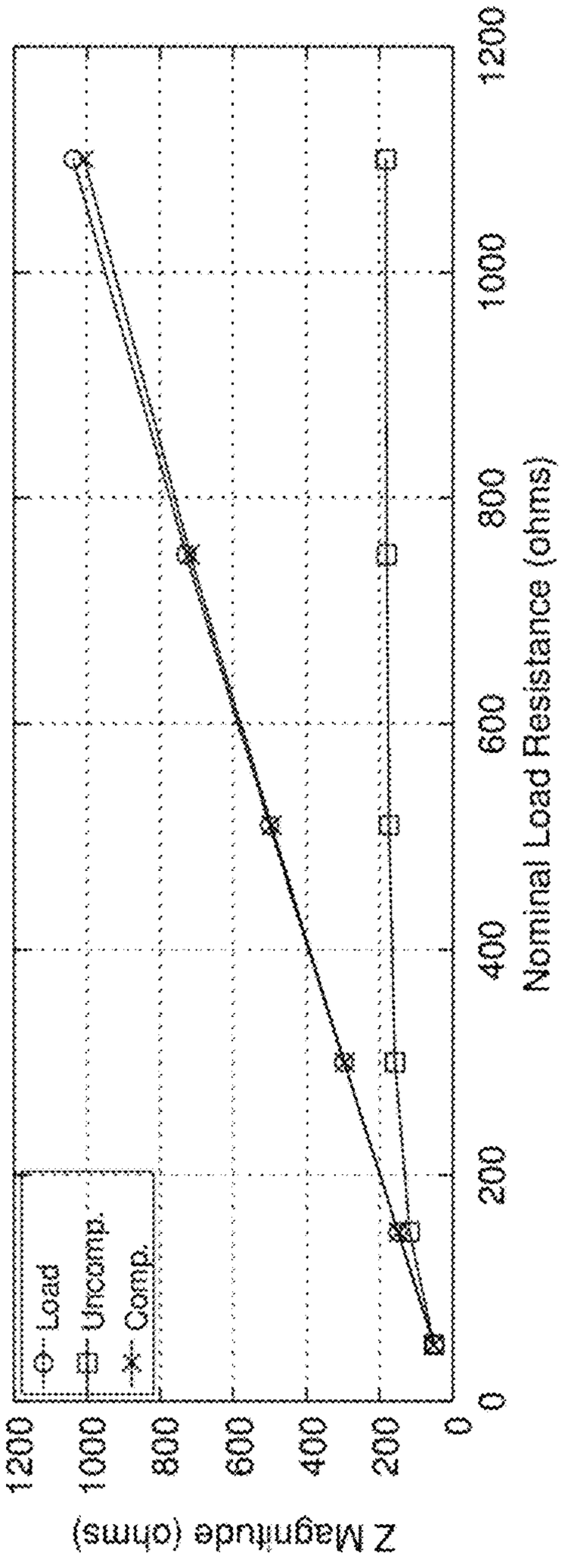
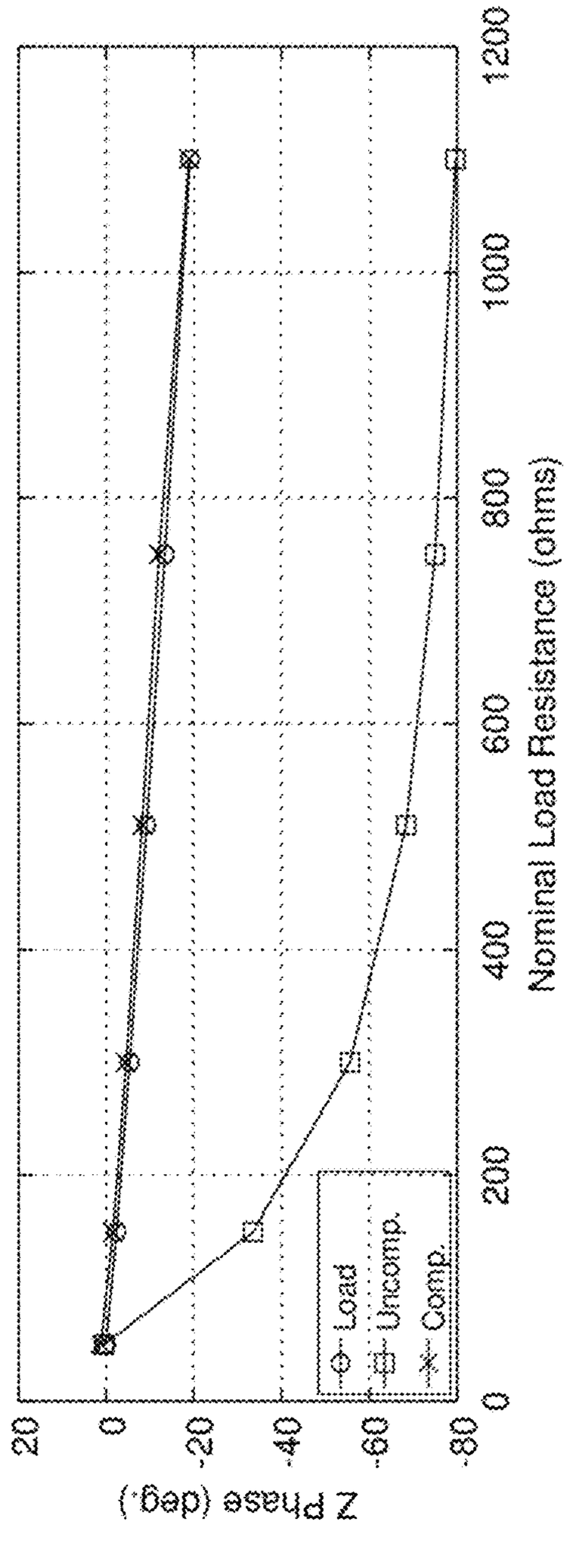


Figure 9b



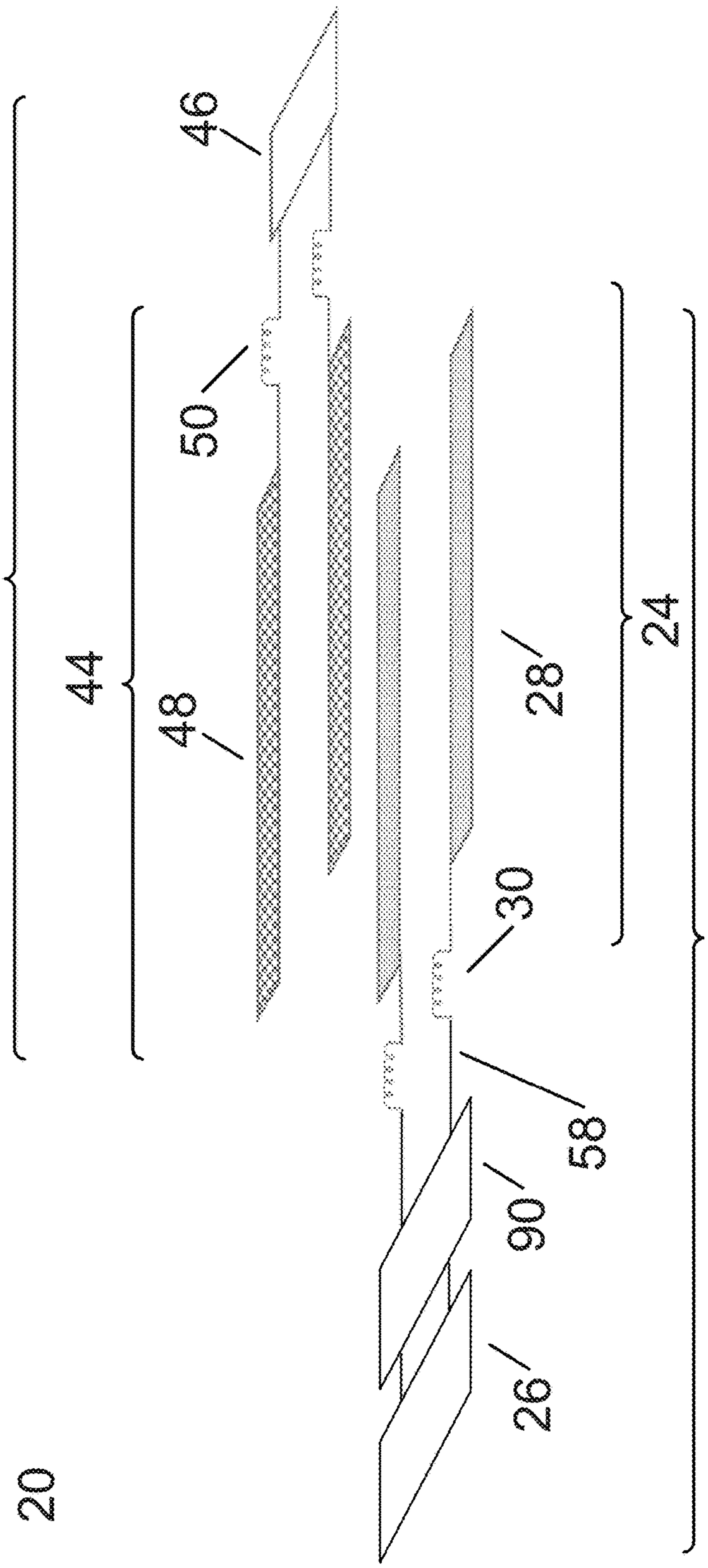


Figure 10

Figure 11a

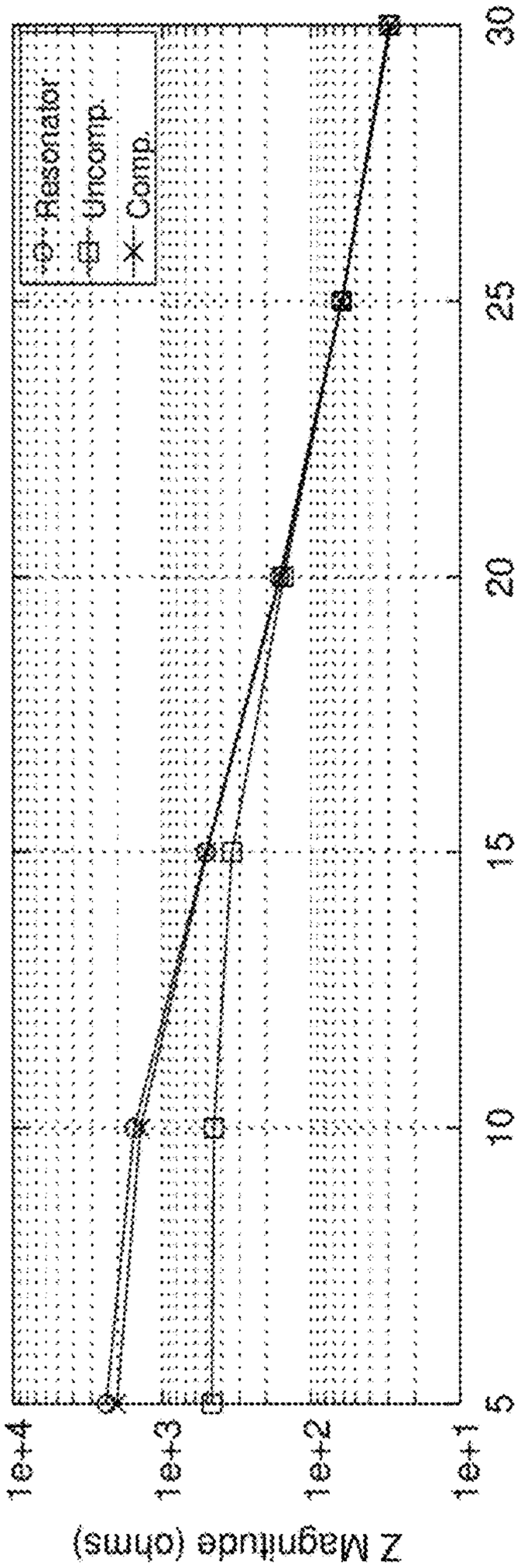
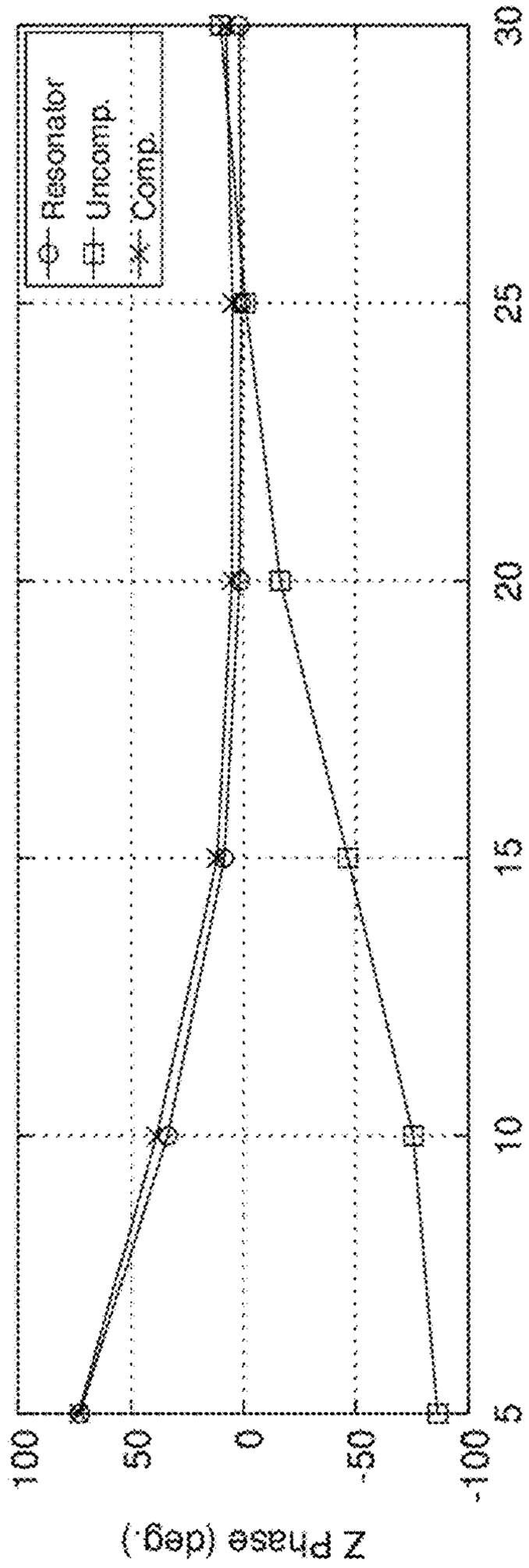


Figure 11b



WIRELESS POWER TRANSFER SYSTEMS, METHODS AND TRANSMITTERS THEREFOR

FIELD

The subject application relates generally to wireless power transfer and in particular, to a wireless power transfer system and transmitter.

BACKGROUND

In the radio frequency (RF) engineering field, the impedance transformation effect results from the connection of a transmission line to a mismatched load impedance. However, it is commonly assumed that the impedance transformation effect is negligible when the transmission line length is small, meaning that the transmission line length is much smaller than a particular wavelength of excitation.

Turning now to FIG. **1**, a diagram of a partial circuit is shown and is generally identified by reference numeral **8**. The circuit **8** has an input impedance Z_{in}, a load impedance Z_L, and a transmission line with a characteristic impedance Z_0, a phase constant β and a physical length Δz. The circuit **8** is driven by a sinusoidal power signal from a power source at a particular wavelength of excitation. The transmission line is assumed to be effectively lossless. The relationship between the input impedance Z_{in} and the load impedance Z_L is expressed as:

$$Z_{in} = Z_0 \frac{Z_L + jZ_0\tan(\beta\Delta z)}{Z_0 + jZ_L\tan(\beta\Delta z)} \quad \text{(Equation 1)}$$

As mentioned above, for electrically small transmission lines the impedance transformation effect is commonly assumed to be negligible. This is consistent with assuming that for electrically small transmission lines the terms involving $\beta\Delta z$ approach zero. The transmission line of FIG. **1** is electrically small when the length of the transmission line is much smaller than the wavelength of excitation of the power signal. When the terms involving $\beta\Delta z$ are assumed to be zero the relationship between the input impedance Z_{in} and the load impedance Z_L reduces to:

$$Z_{in} \cong Z_L \quad \text{(Equation 2)}$$

The accuracy of Equation 2 depends strongly on the magnitude of Z_L relative to Z_0. For electrically small transmission lines $\tan(\beta\Delta z)$ is assumed to be approximately equal to $\beta\Delta z$ as per the small angle approximation. The relationship between the input impedance Z_{in} and the load impedance Z_L can then be expressed as:

$$Z_{in} \cong Z_0 \frac{Z_L + jZ_0\beta\Delta z}{Z_0 + jZ_L\beta\Delta z} \quad \text{(Equation 3)}$$

When $|Z_L|\beta\Delta z << |Z_0|$, Equation 3 can be simplified to the following:

$$Z_{in} \cong Z_L + jZ_0\beta\Delta z \quad \text{(Equation 4)}$$

In this scenario, the delay associated with the transmission line causes a shift in the wavelengths-toward-generator, or clockwise direction around the Smith chart. Since $|Z_L| << |Z_0|$, the result is an effective series inductance.

When $|Z_L| >> |Z_0\beta\Delta z$, Equation 3 can be simplified to the following:

$$Z_{in} \cong \frac{Z_L}{1 + jZ_LY_0\beta\Delta z} \quad \text{(Equation 5)}$$

In Equation 5, Y_0 is Z_0^{-1}. In this scenario, the delay associated with the transmission line causes a shift in the wavelengths-toward-generator, or clockwise direction around the Smith chart. Since $|Z_L| >> |Z_0|$, the result is an effective parallel capacitance. In both Equations 4 and 5, the transmission line causes a phase delay in the circuit **8**.

This phase delay may be problematic for RF systems in which the goal is to efficiently deliver power to a variable load impedance, such as wireless power transfer systems that transfer power via resonant coupling. In such a wireless power transfer system, a power source outputs a sinusoidal power signal that drives a transmit resonator which transfers power to a receive resonator. The system input impedance exhibits an inverse relationship with the distance between the transmit and receive resonators. For a distance of 10 cm to 30 cm, a system input impedance of 1100Ω to 20Ω may be observed.

Table 1 below is an example set of input impedance Z_{in} values calculated for a given set of load impedance Z_L values for a Δz=0.3048 m (12") transmission line. In this example, the load impedance Z_L values are selected from the set including 20Ω, 50Ω, 150Ω, 300Ω, 510Ω, 750Ω and 1100Ω. The characteristic impedance Z_0 of the transmission line is 50Ω. The phase constant β of the transmission line is 0.426 rad/m. Assuming a propagation velocity of ⅔ of the speed of light, this phase constant β corresponds to an excitation frequency of 13.56 MHz. As previously stated, the physical length Δz of the transmission line is 0.3048 m (12"). The wavelength λ of the power signal from the power source is 14.6304 m (576"). Thus, the physical length Δz of the transmission line is $\lambda/48$. Given these values, Z_{in} can be calculated according to Equation 1 for the various Z_L values in the given range.

TABLE 1

Z_L (Ω)	Z_{in} (Ω)
20	20 + j6
50	50 + j0
150	132 − j45
300	189 − j142
510	187 − j242
750	158 − j302
1100	121 − j341

As shown in Table 1, the presence of the transmission line has an impact on the input impedance Z_{in} of the circuit **8** except for the matched impedance case in which the load impedance Z_L is equal to the characteristic impedance Z_0 of the transmission line. In the matched impedance case, both the load impedance Z_L and the input impedance Z_{in} are 50Ω. In the other cases (i.e. when the characteristic and load impedances are mismatched), the presence of the transmission line adds an additional reactance to the input impedance Z_{in}. Thus, the load impedance Z_L and the input impedance Z_{in} are not equal. For example when the load impedance Z_L is 300Ω, the input impedance Z_{in} is 189−j142Ω.

In the context of antennas or resonators in wireless power transfer systems that transfer power via resonant coupling, the additional reactance is analogous to detuning the trans-

mitter of the wireless power transfer system, resulting in less efficient power transfer. It is therefore an object of an aspect of the subject disclosure to at least partially mitigate the impedance transformation effect of transmission lines on electrical systems.

SUMMARY

Accordingly, in one aspect there is provided a transmitter comprising: a transmit resonator electrically connected to a power source of the transmitter via an electrically small transmission line, the transmit resonator comprising: at least two electrodes; and at least one inductive coil electrically connected to the electrodes, wherein the transmit resonator is configured to generate an electric field when excited by the power source, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a power signal generated by the power source; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line, wherein the MMTL has a negative phase velocity (NPV).

In one or more embodiments, the electrically small transmission line has a phase constant β, a physical length Δz, and $\tan(\beta\Delta z)\cong\beta\Delta z$.

In one or more embodiments, the electrically small transmission line has a phase constant β and a physical length Δz, and $\beta\Delta z<0.2$ radians.

In one or more embodiments, the electrically small transmission line has a physical length Δz, the power signal has a wavelength λ, and $\Delta z<\lambda/30$.

In one or more embodiments, the MMTL comprises a unit cell. In another embodiment, the unit cell comprises a symmetrical T-network. In another embodiment, the symmetrical T-network comprises two series capacitors and a shunt inductor. In another embodiment, the capacitance of the capacitors is given by $2C_0$ and the inductance of the inductor is given by L_0, wherein:

$$C_0 \cong \frac{1}{\omega^2 Z_0 \beta \Delta z}$$

$$L_0 \cong \frac{Z_0}{\omega^2 \beta \Delta z}$$

and wherein Z_0 is the characteristic impedance of the transmission line, β is the phase constant of the electrically small transmission line, Δz is the length of the electrically small transmission line and w is the angular frequency of a power signal output by the power source.

In another embodiment, the unit cell comprises a symmetrical π-network.

In one or more embodiments, the MMTL comprises n unit cells, where n is a positive integer.

In one or more embodiments, the transmitter further comprises the power source.

According to another aspect there is provided a system comprising: a transmitter comprising: a transmit resonator electrically connected to a power source of the transmitter via an electrically small transmission line, the transmit resonator comprising: at least two active electrodes; and at least one inductive coil electrically connected to the active electrodes, wherein the transmit resonator is configured to generate an electric field when excited by the power source, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a power signal generated by the power source; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line, wherein the MMTL has a negative phase velocity (NPV); and a receiver configured to extract power from the generated electric field via resonant electric field coupling.

In another embodiment, the receiver further comprises: a load; and a receive resonator electrically connected to the load.

In another embodiment, the receive resonator comprises: at least two active electrodes; and at least one inductive coil electrically connected to the active electrodes of the receiver, wherein the receive resonator is configured to receive power transmit resonator via resonant electric field coupling.

In another embodiment, the transmitter further comprises the power source.

According to another aspect there is a system comprising: a power source; a load electrically connected to the power source via an electrically small transmission line, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a sinusoidal power signal generated by the power source; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line, wherein the MMTL has a negative phase velocity (NPV).

According to another aspect there is provided a transmitter comprising: a transmit resonator electrically connected to a power source of the transmitter via an electrically small transmission line shorter in length than a wavelength of excitation of a power signal generated by the power source, wherein the transmit resonator is configured to generate an electric field when excited by the power; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an impedance transformation caused by the electrically small transmission line.

According to another aspect there is provided a metamaterial transmission line (MMTL) electrically connected between a power source and an electrically small transmission line connected to a load, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a sinusoidal power signal generated by the power source, wherein the MMTL configured to counteract an effect of the electrically small transmission line.

In another embodiment, the MMTL causes a negative phase delay that counters a phase delay caused by the electrically small transmission line.

In another embodiment, the MMTL causes a phase shift that is opposite in sign to a phase shift caused by the electrically small transmission line.

In another embodiment, phase and group velocities of a wave travelling through the MMTL are opposite in sign to phase and group velocities of a wave travelling through the electrically small transmission line.

In another embodiment, propagation through the MMTL incurs a negative phase delay.

In another embodiment, the MMTL has impedance characteristics that are substantially inverse to impedance characteristics of the electrically small transmission line.

BRIEF DESCRIPTION OF THE DRAWINGS

Embodiments will now be described more fully with reference to the accompanying drawings in which:

FIG. **1** is a diagram of a partial circuit;

FIG. **2** is a circuit diagram of a power transfer system in accordance with an aspect of the subject disclosure;

FIG. **3** is a circuit topology of a unit cell of a metamaterial transmission line (MMTL) forming a part of the power transfer system of FIG. **2**;

FIG. **4** is a schematic layout of the unit cell of FIG. **3**;

FIG. **5** is a graph of a maximum length of a transmission line of the power transfer system of FIG. **2** with and without the MMTL;

FIG. **6** is a photograph of a physically manufactured unit cell of FIG. **4**;

FIG. **7***a* is a graph of the magnitude of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.1524 m;

FIG. **7***b* is a graph of the phase of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.1524 m;

FIG. **8***a* is a graph of the magnitude of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.3048 m;

FIG. **8***b* is a graph of the phase of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.3048 m; and

FIG. **9***a* is a graph of the magnitude of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.6096 m;

FIG. **9***b* is a graph of the phase of the input impedance versus the nominal load resistance of a system using the unit cell of FIG. **6** for a transmission line with a physical length of 0.6096 m;

FIG. **10** is a schematic layout of a wireless power transfer system in accordance with an aspect of the subject disclosure;

FIG. **11***a* is a graph of the magnitude of the input impedance of a transmit resonator versus the separation between the transmit and receive resonators of the wireless power transfer system of FIG. **10** using the unit cell of FIG. **6** for a transmission line with a physical length of 0.3048 m; and

FIG. **11***b* is a graph of the phase of the input impedance of the transmit resonator versus the separation between the transmit and receive resonators of the wireless power transfer system of FIG. **10** using the unit cell of FIG. **6** for a transmission line with a physical length of 0.3048 m.

DETAILED DESCRIPTION OF EMBODIMENTS

The foregoing summary, as well as the following detailed description of certain examples will be better understood when read in conjunction with the appended drawings. As used herein, an element or feature introduced in the singular and preceded by the word "a" or "an" should be understood as not necessarily excluding the plural of the elements or features. Further, references to "one example" or "one embodiment" are not intended to be interpreted as excluding the existence of additional examples or embodiments that also incorporate the described elements or features. Moreover, unless explicitly stated to the contrary, examples or embodiments "comprising" or "having" or "including" an element or feature or a plurality of elements or features having a particular property may include additional elements or features not having that property. Also, it will be appreciated that the terms "comprises", "has", "includes" means "including by not limited to" and the terms "comprising", "having" and "including" have equivalent meanings. It will also be appreciated that like reference characters will be used to refer to like elements throughout the description and drawings.

As used herein, the terms "adapted" and "configured" mean that the element, component, or other subject matter is designed and/or intended to perform a given function. Thus, the use of the terms "adapted" and "configured" should not be construed to mean that a given element, component, or other subject matter is simply "capable of" performing a given function but that the element, component, and/or other subject matter is specifically selected, created, implemented, utilized, and/or designed for the purpose of performing the function. It is also within the scope of the subject application that elements, components, and/or other subject matter that is described as being adapted to perform a particular function may additionally or alternatively be described as being configured to perform that function, and vice versa. Similarly, subject matter that is described as being configured to perform a particular function may additionally or alternatively be described as being operative to perform that function.

It will be understood that when an element is referred to as being "on," "attached" to, "connected" to, "coupled" with, "contacting," etc., another element, it can be directly on, attached to, connected to, coupled with or contacting the other element or intervening elements may also be present.

Unless defined otherwise, all technical and scientific terms used herein have the same meaning as is commonly understood by one of skill in the art to which the subject disclosure pertains.

In the following, exemplary transmitters and power transfer systems incorporating the same are disclosed. Broadly, the transmitter comprises a power source; a transmit resonator electrically connected to the power source via an electrically small transmission line, the transmit resonator comprising: at least two electrodes; and at least one inductive coil electrically connected to the electrodes, wherein the transmit resonator is configured to generate an electric field when excited by the power source; and a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line. Particular non-limiting examples of the transmitter and systems incorporating the same will now be described.

Turning now to FIG. **2**, a circuit diagram of a power transfer system is shown and is generally identified by reference numeral **10**. The power transfer system **10** comprises a power source **12** electrically connected via a transmission line **14** to a load **16**. The power source **12** is configured to generate a sinusoidal power signal. As previously described, as a result of the impedance transformation effect, the transmission line **14** results in at least a partial impedance transformation. Thus, the impedance presented to the power source **12** differs from the load impedance. A metamaterial transmission line (MMTL) **18** is electrically connected between the power source **12** and the transmission line **14** and is configured to counteract an impedance

transformation caused by the transmission line **14** as will be described. The MMTL **18** has a negative phase velocity (NPV).

As previously discussed, the transmission line **14** causes a delay that may be problematic for the power transfer system **10** in which the goal is efficient delivery of power. The MMTL **18** is configured to counteract the impedance transformation caused by the transmission line **14**. The MMTL **18** is configured to invert the effect of the transmission line **14**. Specifically, the MMTL **18** is configured to create a negative phase delay that counters that phase delay caused by the transmission line **18** as will be described.

As expressed by Equation 4, the presence of the transmission line **14** between the power source **12** and the load **16** may add an effective series inductance to the impedance seen by the power source **12** when the magnitude of the load impedance Z_L is much smaller than the characteristic impedance Z_0 of the transmission line **14**. In this situation, the MMTL **18** is configured to at least partially reduce the series inductance term of Equation 4 when the magnitude of the load impedance Z_L is much smaller than the characteristic impedance Z_0 of the transmission line **14**.

As expressed by Equation 5, the presence of the transmission line **14** between the power source **12** and the load **16** may add an effective parallel capacitance to the impedance seen by the power source **12** when the magnitude of the load impedance Z_L is much larger than the characteristic impedance Z_0 of the transmission line **14**. In this situation, the MMTL **18** is configured to at least partially reduce the parallel capacitance term of Equation 5 when the magnitude of the load impedance Z_L is much larger than the characteristic impedance Z_0 of the transmission line **58**.

As previously stated, the MMTL **18** is configured to have a NPV. Generally, in the field of RF engineering, the term metamaterial (MM) is applied to elements that exhibit a negative permittivity and a negative permeability. A consequence of the negative permittivity and permeability is that the MM has a negative refractive index (NRI). In the context of transmission lines, which guide electromagnetic energy through a one-dimensional medium (e.g. electromagnetic waves only travel in the forward or reverse directions), the MM is one-dimensional. A one-dimensional MM allows for wave travel along a single dimension only. Thus, the MMTL **18** is a one-dimensional NRI metamaterial. A MMTL **18** with a NRI has electric and magnetic fields that form a left-handed triplet with the wave vector. A MMTL **18** with a left-handed triplet is referred to as a left-handed (LH) transmission line.

When an electromagnetic wave propagates through the MMTL **18** with a NRI, phase and group velocities of the wave travelling through the MMTL **18** are opposite in sign to the wave travelling through the transmission line **14**. The NRI of the MMTL **18** therefore results in the negative phase velocity (NPV). The NPV of the MMTL **18** results in a phase shift. The phase shift resulting from the MMTL **18** is opposite in sign to the phase shift caused by the transmission line **14**. Thus, propagation through the metamaterial incurs a negative phase delay.

One of the key assumptions when deriving the NRI property of the MMTL **18** is that there is only a forward travelling wave. This means that the MMTL **18** must either be made up of an infinite series of unit cells or made of a finite number of unit cells which are terminated in the Bloch impedance, which is generally the characteristic impedance of the MMTL **18**, taken at the terminals between the finite unit cells.

Generally, the MMTL **18** is designed such that the Bloch impedance is equal to the characteristic impedance Z_0 of the system to which it will be applied, and the system is designed such that impedance matching is maintained. Thus, there is only a forward travelling wave. In a mismatched case, the load impedance Z_L is arbitrary and is not necessarily impedance-matched to the transmission line **14**. In the mismatched case, there will be a wave reflection at the load leading to both forward and reverse travelling waves. When there are both forward and reverse travelling waves, metamaterial (MM) theory should break down. However, MM theory can still be applied to design the MMTL **18**, which will counteract the impedance transformation effect of the transmission line **14** as long as the length of the transmission line **14** is electrically small. As previously stated, the length of the transmission line **14** is electrically small when the length is shorter than a wavelength of excitation of the sinusoidal power signal generated by the power source **12**.

The input impedance Z_{in} and the load impedance Z_L of the power transfer system **10** can be represented by the following matrix:

$$\begin{bmatrix} V_{in} \\ I_{in} \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} \cos(\beta\Delta z) & jZ_0\sin(\beta\Delta z) \\ jY_0\sin(\beta\Delta z) & \cos(\beta\Delta z) \end{bmatrix} \begin{bmatrix} V_L \\ I_L \end{bmatrix} \quad \text{(Equation 6)}$$

The ABCD matrix represents the MMTL **18**. The adjacent matrix containing trigonometric functions represents the transmission line **14** and is derivable from Equation 1. V_{in} and I_{in} represent the input voltage and current, respectively, of the power signal generated by the power source **12** such that $Z_{in}=V_{in}/I_{in}$. Similarly, V_L and I_L represent the load voltage and current, respectively, such that $Z_L=V_L/I_L$. To counteract the effects of the transmission line **14**, V_{in} must equal V_L and I_{in} must equal I_L. The nontrivial solution requires the ABCD matrix to be the inverse of the transmission line matrix. Therefore, the ABCD matrix is represented by the following matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} \cos(\beta\Delta z) & -jZ_0\sin(\beta\Delta z) \\ -jY_0\sin(\beta\Delta z) & \cos(\beta\Delta z) \end{bmatrix} \quad \text{(Equation 7)}$$

When the physical length of the transmission line **14** is electrically small Equation 7 can be simplified. In the context of this derivation, the transmission line **14** is electrically small when $\beta\Delta z<0.2$ radians or when the physical length of the transmission line **14** is less than $\lambda/30$. As previously stated, λ is the wavelength of the power signal from the power source **12**. Equation 7 can therefore be simplified using small-angle approximations to the following matrix:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1-\frac{(\beta\Delta z)^2}{2} & -jZ_0\beta\Delta z \\ -jY_0\beta\Delta z & 1-\frac{(\beta\Delta z)^2}{2} \end{bmatrix} \quad \text{(Equation 8)}$$

The transmission line **14** can be represented using the lumped element model. Since the transmission line **14** is assumed to be effectively lossless, the characteristic impedance Z_0 and the phase constant β can be expressed as:

$$Z_0 = \sqrt{\frac{L'}{C'}} \quad \text{(Equation 9)}$$

$$\beta = \omega\sqrt{L'C'} \quad \text{(Equation 10)}$$

The variable ω is the angular frequency of the power signal. The variable L' is the per-unit-length inductance of the transmission line **14**. The variable C' is the per-unit-length capacitance of the transmission line **14**.

Substituting Z_0 and β in Equation 8 for the values in Equations 9 and 10 and recalling that Y_0 is Z_0^{-1} yields the following result:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} = \begin{bmatrix} 1 - \frac{\omega^2 L' C' \Delta z^2}{2} & -j\omega L' \Delta z \\ -j\omega C' \Delta z & 1 - \frac{\omega^2 L' C' \Delta z^2}{2} \end{bmatrix} \quad \text{(Equation 11)}$$

When the MMTL **18** is represented by the matrix identified in Equation 11, the MMTL **18** will counteract the impedance transformation caused by the transmission line **14**. Specifically, the MMTL **18** will counteract the impedance transformation caused by the transmission line **14** on the load impedance.

In one embodiment, the MMTL **18** comprises a single unit cell implemented as a symmetrical T-network. The circuit topology of an exemplary symmetrical T-network is shown in FIG. **3**. While the circuit topology shown in FIG. **3** is associated with a small propagation delay, the propagation delay is assumed to be negligible for this analysis. For the circuit topology shown in FIG. **3**, the parameters of the ABCD matrix of the MMTL **18** can be expressed as follows:

$$\begin{bmatrix} A & B \\ C & D \end{bmatrix} \cong \begin{bmatrix} 1 + \frac{ZY}{2} & Z + \frac{Z^2 Y}{4} \\ Y & 1 + \frac{ZY}{2} \end{bmatrix} \quad \text{(Equation 12)}$$

The A, C and D parameters in Equation 11 can be achieved if:

$$Y = -j\omega C' \Delta z \quad \text{(Equation 13)}$$

$$Z = -j\omega L' \Delta z \quad \text{(Equation 14)}$$

Based on these results, the parameter for B is given by:

$$B = -j\omega L' \Delta z \left(1 - \frac{\omega^2 L' C' \Delta z^2}{4}\right) \quad \text{(Equation 15)}$$

Alternatively, this can be written as:

$$B = -j\omega L' \Delta z \left(1 - \left(\frac{\beta \Delta z}{2}\right)^2\right) \quad \text{(Equation 16)}$$

As previously stated, it has been assumed that the transmission line **14** is electrically small and thus $\beta\Delta z < 0.2$ radians. The term $1-(\beta\Delta z/2)^2$ in Equation 16 will therefore be in the range of 0.99 to 1. Thus, $1-(\beta\Delta z/2)^2$ can be approximated to one. Accordingly, Equation 16 can be approximated as:

$$B \cong -j\omega L' \Delta z \quad \text{(Equation 17)}$$

The T-network is implemented using two series capacitors, each with a value of $2C_0$, and a shunt inductor with a value of L_0. These can be expressed as:

$$C_0 \cong \frac{1}{\omega^2 L' \Delta z} \quad \text{(Equation 18)}$$

$$L_0 \cong \frac{1}{\omega^2 C' \Delta z} \quad \text{(Equation 19)}$$

Thus, the T-network shown in FIG. **3** can be designed to satisfy three of the four ABCD parameters for inverting the effects of an electrically small transmission line, and to approximate the fourth parameter (B) very closely.

C_0 and L_0 can also be expressed in terms of the parameters of the transmission line **14**. As previously stated, the parameters comprise the characteristic impedance Z_0, the phase constant β and the physical length Δz of the transmission line **14**. Expressing Equations 18 and 19 in terms of the parameters of the transmission line **14** yields the following:

$$C_0 \cong \frac{1}{\omega^2 Z_0 \beta \Delta z} \quad \text{(Equation 20)}$$

$$L_0 \cong \frac{Z_0}{\omega^2 \beta \Delta z} \quad \text{(Equation 21)}$$

Equations 20 and 21 are the same as those which may be used to design a MMTL comprising a single cell in an impedance matched system. Thus, MM theory can still be applied for an impedance mismatched system when the transmission line **14** is electrically small.

Inserting C_0 and L_0 into the circuit topology of FIG. **3** yields the resultant single unit cell of the MMTL **18** as shown in FIG. **4**. In this embodiment, the single unit cell comprises a first capacitor **62** connected in a series to a second capacitor **64** and a shunt inductor **66** between the capacitors **62** and **64**. The capacitance of the first and second capacitors **62** and **64** is given by $2C_0$, where C_0 has a value as per Equation 18. The inductance of the inductor **66** is given by L_0 with a value as per Equation 19.

While the single unit cell of the MMTL **18** has been described as a symmetrical T-network, one of ordinary skill in the art will appreciate that other configurations are possible. For example, in another embodiment, the single cell of the MMTL **18** may be a symmetrical w-network.

Furthermore, while MMTL **18** comprising a single unit cell has been described, one of ordinary skill in the art will appreciate that still other configurations are possible. For example, in another embodiment, the MMTL **18** comprises n unit cells, where n is an integer value and greater than 1. In this embodiment, each unit cell is configured to produce a fraction of the negative phase delay required to cancel the phase delay of the transmission line **14**. In particular, each unit cell is configured to cancel an equal fraction of the phase delay of the transmission line **14**. Thus, each unit cell is configured to cancel a phase delay of $\beta\Delta z/n$. To calculate the values of the components of each unit cell, Equations 18 to 21 are multiplied by n.

The MMTL **18** comprising a single unit cell as described above corrects for the impedance transformation effect through a range of lengths Δz of the transmission line **14**. The maximum length Δz of the transmission line **14** at which the MMTL **18** no longer corrects for the impedance transformation effect occurs when the input impedance Z_{in} is no longer sufficiently close to the load impedance Z_L. For this disclosure, the input impedance Z_{in} is no longer sufficiently close to the load impedance Z_L when the relative magnitude error between the input impedance Z_{in} and the load impedance Z_L exceeds 10% or the absolute phase error between the input impedance Z_{in} and the load impedance Z_L exceeds 5°. With the application of the MMTL **18**, the upper limit on the applicable physical length Δz of the transmission line **14** is dictated by the assumption that the transmission line **14** is electrically small. As the physical length Δz increases to the point where the transmission line **14** is no longer electrically small, MM theory breaks down. In this case, the MMTL **18** no longer counteracts the impedance transformation effect of the transmission line **14**. There is also a lower limit on the physical length Δz of the transmission line **14** for which use of the MMTL **18** is of value. As the physical length Δz of the transmission line **14** decreases toward zero, eventually the impedance transformation effect becomes negligible and compensation from the MMTL **18** is not necessary. These two considerations define the range of physical lengths Δz of the transmission line **14** over which it is desirable to apply the MMTL **18**.

Compensated and un-compensated power transfer systems were modeled using Octave™ to determine maximum lengths Δz of the transmission line **14**. When the MMTL **18** is present such as in the power transfer system **10**, the power transfer system **10** is referred to as a compensated system. When the MMTL **18** is not present in the power transfer system **10**, the power transfer system is referred to as an un-compensated system.

A characteristic impedance Z_0 of 50Ω and a phase constant β of 0.426 rad/m were utilized for the calculations. Assuming a propagation velocity of ⅔ of the speed of light, this phase constant β corresponds to an excitation frequency of 13.56 MHz. Calculations were made for load impedances Z_L with phase of −45°, 0° and +45°. The results of these calculations are shown in FIG. **5**, where FIG. **5** is a graph of the maximum length Δz of the transmission line **14** versus the load impedance Z_L magnitude for the compensated system and the un-compensated system.

As will be appreciated, except when the load impedance Z_L is equal to the characteristic impedance Z_0 of the transmission line **14** (i.e. the matched case), the maximum length Δz of the transmission line **14** is much lower in the un-compensated system than in the compensated system. Based on the results shown in FIG. **5**, the maximum lengths Δz of the transmission line **14** for a sample set of load impedance magnitudes $|Z_L|$ is reproduced in Table 2. Note that the maximum Δz values in Table 2 were obtained by averaging the results for the three different load impedance phases −45°, 0° and +45°.

TABLE 2

$\|Z_L\|$ (Ω)	Uncompensated System - Maximum Δz (m)	Compensated System - Maximum Δz (m)
5	0.027	1.360
50	Infinite	N/A (matched)
100	0.135	1.630
200	0.069	1.320
500	0.027	0.978
1000	0.014	0.777

As shown in Table 2, the presence of the MMTL **18** in the compensated system allows for a greater maximum length Δz of the transmission line **14** compared to when the MMTL **18** is not present in the un-compensated system. For example, when the load impedance Z_L is 200Ω the maximum length Δz of the transmission line **14** in the un-compensated system (i.e. without the MMTL **18**) is 0.069 m, but the maximum length Δz of the transmission line **14** in the compensated system (i.e. with the MMTL **18**) is 1.32 m. This represents an increase by a factor of over 19.

Thus, in the power transfer system **10**, the maximum length Δz of the transmission line **14** for which the input impedance Z_{in} is sufficiently close to the load impedance Z_L is greater than the maximum length Δz of the transmission line **14** when the MMTL **18** is not present. The increased maximum length Δz of the transmission line **14** of the power transfer system **10** that comprises the MMTL **18** allows for further separation of the power source **12** from the load **16** while at least partially reducing or eliminating the impedance transformation effect of the transmission line **14**. This also allows for further separation of other power electronics from the load **14**.

Turning now to FIG. **6**, a physically manufactured unit cell is shown and is generally identified at reference character **70**. The unit cell **70** is designed and fabricated according to the circuit diagram of FIG. **4**. The unit cell **70** comprises a printed circuit board (PCB) **72** upon which a first capacitor **74**, a second capacitor **76** and an inductor **78** are mounted. The unit cell **70** further comprises two connectors **80** and **82**. Each connector **80** and **82** is configured to connect to either the transmission line **14** or the power source **12**.

The unit cell **70** parameters were calculated such that the unit cell **70** is configured to counteract the combined impedance transformation caused by a transmission line **14**, the PCB **72**, the first connector **80** and the second connector **82**. In this embodiment, the transmission line **14** is a coaxial SMA cable. Calculations for the parameters of the unit cell **70** were performed for transmission line lengths of 0.1524 m (6"), 0.3048 m (12") and 0.6096 m (24"). In this embodiment, the operating frequency of the power transfer system is 13.56 MHz. The characteristic impedance Z_0 is 50Ω. The phase constant β is 0.426 rad/m. The required values of the first capacitor **74**, second capacitor **76** and inductor **78** were determined according to Equations 20 and 21. For these calculations, Δz represents the length of the transmission line **14** in addition to the combined effective length of the PCB **72**, first connector **80** and second connector **82** of the unit cell **70**. As previously stated with reference to FIG. **4**, the required capacitance of the first and second capacitors **74** and **76** is given by $2C_0$ while the required inductor **78** is given by L_0. The required values of C_0 and L_0 are shown in Table 3 below. While fabricating the unit cell **70**, the particular capacitances and inductances calculated according to Equations 20 and 21 were approximated as closely as possible using available components. As one of ordinary skill in the art will appreciate, using components which do

not closely match the calculated capacitances and inductances may result in a unit cell **70** which does not function as required.

TABLE 3

Cable Length (m)	Calculated L_0 (μH)	Calculated C_0 (nF)
0.1524	6.73	2.69
0.3048	3.89	1.55
0.6096	2.14	086

A set of constructed unit cells **70** having the parameters shown in Table 3 were tested for transmission lines **14** with length Δz of 0.1524 m (6"), 0.3048 m (12") and 0.6096 (24"). Each unit cell **70** was tested for a series of radio frequency (RF) load resistors with following nominal values: 50Ω, 150Ω, 300Ω, 510Ω, 750Ω and 1100Ω. A non-negligible impedance transformation was observed when measuring the resistors directly using a vector network analyzer (VNA) as shown in Table 4. These measured results were taken as the load impedances Z_L.

TABLE 4

Nominal Load Resistance (Ω)	Measured Z_L (Ω)
50	49.9 + j0.1
150	149.4 – j6.2
300	296.4 – j27.8
510	495.3 – j80.5
750	708.5 – j168.6
1100	980.6 – j340.9

The transmission line **14** with the length Δz of 0.1524 m (6") was connected between the power source **12**, which in this case was the VNA, and the particular load resistor in the set. The associated input impedance was then recorded on the VNA for a system that did not include the unit cell **70** (i.e. the uncompensated system).

In the compensated system, the unit cell **70** with the parameters shown in Table 3 designed to counteract the impedance transformation caused by the transmission line **14** with the physical length Δz of 0.1524 m (6") was connected between the power source **12**, which in this case was the VNA, and the transmission line **14**. The input impedance for this compensated system was then recorded.

FIG. **7***a* is a graph of the magnitude of the input impedance versus the nominal load resistance for the case of the transmission line **14** with the physical length Δz of 0.1524 m (6"), a characteristic impedance Z_0 of 50Ω and a phase constant β of 0.426 rad/m are shown. The input impedance magnitude for the uncompensated system is identified as 'Uncomp' in the legend. The load impedance magnitude is identified as 'Load' in the legend. As shown in FIG. **7***a*, the 'Uncomp' data does not match the 'Load' data. For example, when the nominal load resistance is 1100Ω, the magnitude of the input impedance for the uncompensated case is less than 600Ω. The magnitude of the input impedance for the compensated system when the unit cell **70** is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, closely tracks the 'Load' data.

FIG. **7***b* is a graph of the phase of the input impedance Z versus the nominal load resistance. The phase of the input impedance for the uncompensated system is identified as 'Uncomp' in the legend. The phase obtained when directly measuring the load is identified as 'Load' in the legend. As shown in FIG. **7***b*, comparing the 'Uncomp' data to the 'Load' data clearly shows a difference that rises as high as 40° The phase of the input impedance for the compensated system when the unit cell **70**, is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, is almost equal to the 'Load' data.

Thus, the unit cell **70** at least partially counteracts the impedance transformation caused by the 0.1524 m (6") transmission line **58**.

In another unit cell **70** of the set, the constructed unit cell **70** was tested for the transmission line **14** with the length Δz of 0.3048 m (12"). The transmission line **14** was connected between the power source **12**, which in this case was the VNA, and the particular load resistor. The associated input impedance Z_{in} was then recorded for a system that did not include the unit cell **70** (i.e. the uncompensated system).

In the compensated system, the unit cell **70** with the parameters shown in Table 3 designed to counteract the impedance transformation caused by the transmission line **14** with the physical length Δz of 0.3048 m (12") was connected between the power source **12**, which in this case was the VNA, and the transmission line **14**. The input impedance Z_{in} for this compensated system was then recorded.

FIG. **8***a* is a graph of the magnitude of the input impedance versus the nominal load resistance for the case of the transmission line **14** with the physical length Δz of 0.3048 m (12"), a characteristic impedance Z_0 of 50Ω and a phase constant β of 0.426 rad/m are shown. The input impedance magnitude for the uncompensated system is identified as 'Uncomp' in the legend. The load impedance magnitude is identified as 'Load' in the legend. As shown in FIG. **8***a*, the 'Uncomp' data does not match the 'Load' data. For example, when the nominal load resistance is 1100Ω, the magnitude of the input impedance for the uncompensated system is less than 400Ω. This results in a difference of 800Ω which is greater than the 500Ω difference recorded for a transmission line **58** with the physical length Δz of 0.1524 m (6"). The input impedance magnitude for the compensated system when the unit cell **70** is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, closely tracks the 'Load' data.

FIG. **8***b* is a graph of the phase of the input impedance Z versus the nominal load resistance. The phase of the input impedance for the uncompensated system is identified as 'Uncomp' in the legend. The phase obtained when directly measuring the load is identified as 'Load' in the legend. As shown in FIG. **8***b*, comparing the 'Uncomp' data to the 'Load' data shows a difference that rises as high as about 50°. The phase of the input impedance for the compensated system, when the unit cell **70**, is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, is almost equal to the 'Load' data. The 'Comp' data is within 5° of the 'Load' data.

Thus, the unit cell **70** at least partially counteracts the impedance transformation caused by the 0.3048 m (12") transmission line **14**.

In another unit cell **70** of the set, the constructed unit cell **70** was tested for the transmission line **14** with the length Δz of 0.6096 m (24"). The transmission line **14** was connected between the power source **12**, which in this case was the VNA, and the load resistor. The associated input impedance Z_{in} was then recorded on the VNA for a system **10** that did not include the unit cell **70** (i.e. the uncompensated system).

In the compensated system, the unit cell **70** with the parameters shown in Table 3 designed to counteract the impedance transformation caused by the transmission line **14** with the physical length Δz of 0.6096 m (24") was connected between the power source **12**, which in this case was the VNA, and the transmission line **14**. The input impedance Z_{in} for this compensated system was then recorded.

FIG. **9***a* is a graph of the magnitude of the input impedance versus the nominal load resistance for the case of the transmission line **14** with the physical length Δz of 0.6096 m (24"), a characteristic impedance Z_0 of 50Ω and a phase constant β of 0.426 rad/m are shown. The input impedance magnitude for the uncompensated system is identified as 'Uncomp' in the legend. The load impedance magnitude is identified as 'Load' in the legend. As shown in FIG. **9***a*, the 'Uncomp' data does not match the 'Load' data. For example, when the nominal load resistance is 1100Ω, the magnitude of the input impedance for the uncompensated system is less than 200Ω. This results in a difference of more than 900Ω which is greater than: the 800Ω difference recorded for a transmission line **14** with the physical length Δz of 0.3048 (12"); and the 500Ω difference recorded for the transmission line **14** with the physical length z of 0.1524 m (6"). The input impedance magnitude for the compensated system when the unit cell **70** is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, closely tracks the 'Load' data.

FIG. **9***b* is a graph of the phase of the input impedance Z versus the nominal load resistance. The phase of the input impedance for the uncompensated system is identified as 'Uncomp' in the legend. The phase obtained when directly measuring the load resistance is identified as 'Load' in the legend. As shown in FIG. **9***b*, comparing the 'Uncomp' data to the 'Load' data clearly shows a difference that rises as high as 600. The phase of the input impedance for the compensated system, when the unit cell **70** is present, is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **70** is present, is almost equal to the 'Load' data. The 'Comp' data is within 50 of the 'Load' data.

Thus, the unit cell **70** at least partially counteracts the impedance transformation caused by the 0.6096 m (24") transmission line **14**.

Turning now to FIG. **10**, a wireless power transfer system is shown and is generally identified by reference numeral **20**. The wireless power transfer system **20** is similar to the power transfer system described in U.S. Pat. No. 9,653,948 issued on May 16, 2017 which is incorporated herein by reference in its entirety. The wireless power transfer system **20** comprises a transmitter **22** comprising a transmit resonator **24** electrically connected via a transmission line **58** to a power source **26**. The wireless power transfer system **20** also comprises a receiver **42** comprising a receive resonator **44** electrically connected to a load **46**. A MMTL is electrically connected between the power source **26** and the transmission line **58**. The MMTL is configured to counteract the effect of the transmission line **58** as previously described. In this embodiment, the MMTL is a unit cell **90**. The unit cell **90** is similar to the unit cell **70** previously described. The unit cell **90** is designed and fabricated according to the circuit diagram of FIG. **4**. The unit cell **90** comprises a printed circuit board (PCB) upon which a first capacitor, a second capacitor and an inductor are mounted. The unit cell **90** further comprises two connectors. The unit cell **90** parameters were calculated such that the unit cell **90** is configured to counteract the combined impedance transformation caused by the transmission line **58**, the PCB, and the first and second connectors. The first connector is configured to connect to the power source **26**. The second connect is configured to connect to the transmission line **58**.

The transmit resonator **24** comprises two electrodes **28**, forming a capacitor. Each electrode **28** is electrically connected to an inductive coil **30** that is electrically connected via the transmission line **58** and the unit cell **90** to the power source **26**. In this embodiment, the electrodes **28** of the transmit resonator **24** are laterally spaced. The electrodes **28** are elongate plates. The major surfaces of the elongate plates are in the same plane. Each elongate plate is a generally rectangular, planar plate. Each elongate plate is formed of electrically conductive material. While two electrodes **28** are depicted, one of ordinary skill in the art will appreciate that more electrodes may be used.

The receive resonator **44** comprises two electrodes **48**, forming a capacitor, electrically connected to an inductive coil **50**. In this embodiment, the electrodes **48** of the receive resonator **44** are laterally spaced. The electrodes **48** are elongate plates. The major surfaces of the elongate plates are in the same plane. Each elongate plate is a generally rectangular, planar plate. Each elongate plate is formed of electrically conductive material. While two electrodes **48** are depicted, one of ordinary skill in the art will appreciate that more electrodes may be used.

In this embodiment, the inductive coils **30** and **50** of the transmit and receive resonator **24** and **44**, respectively, are ferrite core inductors. One of skill in the art will appreciate that other cores are possible. The receive resonator **44** is tuned to the resonant frequency of the transmit resonator **24**.

While the electrodes **28** and **48** have been described as laterally spaced, elongate plates, one of skill in the art will appreciate that other configurations are possible including, but not limited to, concentric, coplanar, circular, elliptical, disc, etc., electrodes. Other suitable electrode configurations are described in U.S. patent application Ser. No. 14/846,152 filed on Sep. 4, 2015, the relevant portions of which are incorporated herein by reference.

While two inductive coils **30** have been shown as being connected via the transmission line **58** and the unit cell **90** in series to the power source **26**, one of skill in the art will appreciate that a single inductive coil **30** may be used for both electrodes **28** of the transmit resonator **24**. In this case, the single inductive coil **30** is electrically connected via the transmission line **58** and the unit cell **90** to the power source **26** and the electrodes **28** in parallel.

Similarly, while two inductive coils **30** have been shown as being connected in series to the load **46**, one of skill in the art will appreciate that a single inductive coil **30** may be used for both electrodes **48** of the receive resonator **44**. In this embodiment, the single inductive coil **30** is electrically connected to the load **46** and the electrodes **48** in parallel.

During operation, power is transferred from the power source **26** to the electrodes **28** via the unit cell **90**, the transmission line **58** and the inductive coils **30** of the transmit resonator **24**. In particular, the sinusoidal power signal from the power source **26** that is transmitted to the electrodes **28** via the inductive coil **30** excites the transmit resonator **24** causing the transmit resonator **24** to generate an electric field. A differential voltage is applied on the electrodes **28** of the transmit resonator **24**, which creates an electric field in the surrounding environment. When the receive resonator **44** is placed within the generated electric field and tuned to the resonant frequency of the transmit resonator **24**, the receive resonator **44** extracts power from

the transmitter **22** via resonant electric field coupling. The extracted power is then transferred from the receive resonator **44** to the load **46**.

During operation the transmit resonator **24** may generate a magnetic field. However, little, if any power is transferred from the transmit resonator **24** to the receive resonator **44** via magnetic field coupling as the receive resonator **44** is not configured to receive power via magnetic field coupling.

The transmission line **58** separates the power source **26** from the transmit resonator **24**. The transmission line **58** may further separate the transmit resonator **24** from other transmit electronics which are not shown. The transmission line **58** results in at least a partial impedance transformation which adds reactance and changes the resistance of the impedance presented to the power source **26** as previously described.

The input impedance of the load which effectively consists of the transmit resonator **24** is designed to equal a nominal value to which the power source **26** can most efficiently and effectively transfer power. In conventional wireless power transfer systems, this is achieved using an impedance transformation network at the power source **26**. However, if the impedance of the transmit resonator **24** varies over time due to, for example, changes in the distance between the transmitter **22** and the receiver **42**, the impedance transformation network must be dynamic.

The presence of the transmission line **58** affects the impedance seen by a dynamic (or static) impedance transformation network (as shown in Table 1 and Equations 4 and 5). This effectively detunes the transmit resonator **24** from the resonant frequency. Thus, the impedance transformation network is required to correct for the affected impedance. This places an additional burden on the impedance transformation network. The unit cell **90** addresses this by countering the impedance transformation caused by the transmission line **58**, thus ensuring that the transmit resonator **24** stays tuned.

As previously stated, the input impedance of the transmit resonator **24** acts as the load impedance and varies with the degree of coupling to the receive resonator **44**. In this case, the degree of coupling and therefore the impedance is varied by changing the distance separating the resonators **24** and **44**, respectively. To obtain the baseline measurements, the power source **26** (VNA) was connected directly to the transmit resonator **24** and the receive resonator **44** was terminated in a 50Ω load.

A transmission line **58** with a physical length Δz of 0.3048 m (12"), characteristic impedance Z_0 of 50Ω and phase constant β of 0.426 rad/m was connected between the power source **26** (VNA) and the transmit resonator **24**. The unit cell **90** was connected between the transmission line **58** and the transmit resonator **24**. In an uncompensated system, the unit cell **90** was omitted. The separation between the transmit and receive resonators **24** and **44**, respectively, was varied from 5 to 30 cm in 5 cm steps and the input impedance Z_{in} was recorded for the compensated and uncompensated systems.

FIG. **11***a* is a graph of the magnitude of the input impedance of the transmit resonator **24** versus the separation between the transmit and receive resonators **24** and **44**, respectively. The input impedance magnitude for the uncompensated system is identified as 'Uncomp' in the legend. The resonator impedance magnitude is identified as 'Resonator' in the legend. As shown in FIG. **11***a*, the 'Uncomp' data does not match the 'Resonator' data. For example, at a resonator separation of 10 cm, the magnitude of the input impedance of the transmit resonator should be approximately 1500Ω. However, the uncompensated results indicate an input impedance magnitude of roughly 450Ω. The input impedance magnitude for the compensated system when the unit cell **90** is present is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **90** is present, closely tracks the 'Resonator' data.

FIG. **11***b* is a graph of the phase of the input impedance versus the separation between the transmit and receive resonators **24** and **44**, respectively. The phase of the input impedance for the uncompensated system is identified as 'Uncomp' in the legend. The phase obtained when directly measuring the resonator impedance is identified as 'Resonator' in the legend. As shown in FIG. **11***b*, comparing the 'Uncomp' data to the 'Resonator' data clearly shows a difference that rises as high as approximately 150°. The phase of the input impedance for the compensated system, when the unit cell **90** is present, is identified as 'Comp' in the legend. In contrast with the 'Uncomp' data, the 'Comp' data, where the unit cell **90** is present, is almost equal to the 'Load' data.

Thus, the unit cell **90** at least partially counteracts the impedance transformation effect of the 0.3048 m (12") transmission line **58** on the input impedance of the transmit resonator **24** in the wireless power transfer system **20** when the separation distances between the transmit and receive resonators vary.

Although embodiments have been described above with reference to the figures, those of skill in the art will appreciate that variations and modifications may be made without departing from the scope thereof as defined by the appended claims.

What is claimed is:

1. A transmitter comprising:

a transmit resonator electrically connected to a power source of the transmitter via an electrically small transmission line, the transmit resonator comprising:

at least two electrodes; and

at least one inductive coil electrically connected to the electrodes, wherein the transmit resonator is configured to generate an electric field when excited by the power source, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a power signal generated by the power source; and

a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line, wherein the MMTL has a negative phase velocity (NPV).

2. The transmitter of claim **1**, wherein the electrically small transmission line has a phase constant β and a physical length Δz, and wherein

$$\tan(\beta\Delta z)\approx\beta\Delta z.$$

3. The transmitter of claim **1**, wherein the electrically small transmission line has a phase constant β and a physical length Δz, and wherein $\beta\Delta z<0.2$ radians.

4. The transmitter of claim **1**, wherein the electrically small transmission line has a physical length Δz, wherein the power signal has a wavelength λ, and wherein $\Delta z<\lambda/30$.

5. The transmitter of claim **1**, wherein the MMTL is configured to counteract the impedance transformation effect of the electrically small transmission line on an input impedance of the transmit resonator.

6. The transmitter of claim **1**, wherein the MMTL comprises a unit cell.

7. The transmitter of claim 6, wherein the unit cell comprises a symmetrical T-network.

8. The transmitter of claim 7, wherein the symmetrical T-network comprises two series capacitors and a shunt inductor.

9. The transmitter of claim 8, wherein the capacitance of the capacitors is given by $2C_0$ and the inductance of the inductor is given by L_0, wherein:

$$C_0 \cong \frac{1}{\omega^2 Z_0 \beta \Delta z}$$

$$L_0 \cong \frac{Z_0}{\omega^2 \beta \Delta z}$$

and wherein Z_0 is the characteristic impedance of the transmission line, β is the phase constant of the electrically small transmission line, Δz is the length of the electrically small transmission line and w is the angular frequency of a power signal output by the power source.

10. The transmitter of claim 6, wherein the unit cell comprises a symmetrical π-network.

11. The transmitter of claim 1, wherein the MMTL comprises n unit cells, where n is a positive integer.

12. The transmitter of claim 1, further comprising the power source.

13. A system comprising:

a transmitter comprising:

a transmit resonator electrically connected to a power source of the transmitter via an electrically small transmission line, the transmit resonator comprising:

at least two electrodes; and

at least one inductive coil electrically connected to the electrodes, wherein the transmit resonator is configured to generate an electric field when excited by the power source, wherein the electrically small transmission line is shorter in length than a wavelength of excitation of a power signal generated by the power source; and

a metamaterial transmission line (MMTL) electrically connected between the power source and the electrically small transmission line, the MMTL configured to counteract an effect of the electrically small transmission line, wherein the MMTL has a negative phase velocity (NPV); and

a receiver configured to extract power from the generated electric field via resonant electric field coupling.

14. The system of claim 13, wherein the receiver further comprises:

a load; and

a receive resonator electrically connected to the load.

15. The system of claim 14, wherein the receive resonator comprises:

at least two electrodes; and

at least one inductive coil electrically connected to the electrodes of the receiver, wherein the receive resonator is configured to receive power transmit resonator via resonant electric field coupling.

16. The system of claim 1, wherein the transmitter further comprises the power source.

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